Agnostic Structural Disturbances (ASDs): Detecting and Reducing Misspecification in Empirical Macroeconomic Models

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Abstract

Constructing empirical specifications for structural economic models is difficult, if not impossible. As shown in this paper, even minor misspecifications may lead to large distortions for parameter estimates and implied model properties. We propose a novel concept, namely an agnostic structural disturbance (ASD), that can be used to both detect and correct for misspecification of structural disturbances and is easy to implement. While agnostic in nature, the estimated coefficients and associated impulse response functions of these ASDs allow us to give them an economic interpretation. We adopt the methodology to the Smets-Wouters model and formulate an improved risk-premium and an improved investment-specific productivity disturbance.

Keywords: DSGE, full-information model estimation, structural disturbances

1. Introduction

Exogenous random shocks are the lifeblood of modern macroeconomic business cycle models. They enter the model as innovations to structural disturbances that affect key aspects of the model. Recent generations of business cycle models include a multitude of structural disturbances. Structural disturbances impose restrictions on model equations and,

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thus, on the model's solutions. Therefore, each structural disturbance has to enter *each* model equation correctly. This is a concern, since we often do not have independent evidence on how structural disturbances should affect the system. For example, should a risk-premium disturbance affect all Euler equations or only some? Is it correct to assume that structural disturbances are uncorrelated as is commonly done? Chari et al. (2007) propose "wedges" as alternatives to standard structural disturbances, but wedges also impose restrictions.

The contributions of this paper are threefold. First, we propose the agnostic structural disturbance (ASD) as an alternative type of *structural* disturbance. The procedure simply involves adding structural disturbances with associated reduced-form coefficients to *each* model equation or alternatively to each policy rule. In contrast to regular structural disturbances and wedges, ASDs impose *no* additional restrictions on policy rules. Nevertheless, they are different from measurement error, because they are structural and propagate through the system like regular structural disturbances. The procedure of Cúrdia and Reis (2012) shares with ours the ability to deal with correlated structural disturbances, but their disturbances still impose all the restrictions on model equations of regular structural disturbances.

Our ASD procedure can be used to test whether regular structural disturbances are correctly specified and to enrich an empirical specification by adding ASDs as additional structural disturbances. Using Monte Carlo experiments, we document that the ASD procedure is capable of detecting and correcting for misspecification in samples of typical size.

The second contribution of our paper is to test whether the structural disturbances of the model in Smets and Wouters (2007) (SW) are correctly specified using the same US postwar data set. We find that the risk-premium and the investment-specific productivity disturbance are not. We use our procedure to improve on the SW empirical specification. Our preferred specification (based on marginal likelihood considerations) has three ASDs and excludes the SW risk-premium and the SW investment-specific disturbance.

Although the ASD procedure itself does not rely on any economic reasoning, the estimation results – both the associated coefficients and their impulse response functions (IRFs) – may reveal a lot about the type of structural disturbance the data has identified. For example, we interpret one of the ASDs in our adjusted empirical specification of the SW model as an "investment-modernization" disturbance, because it stimulates investment, but at the same time leads to faster depreciation of the existing capital stock. The second ASD of our empirical model has features in common with both a risk-premium and a preference disturbance but is also different from both. Finally, the third ASD captures increases in the wage mark-up disturbance that are associated with an increase in the utilized capital stock.

The third contribution of our paper consists of showing that *minor* misspecifications of the empirical model regarding structural disturbances can easily lead to *large* distortions for parameter estimates and model properties, such as business cycle statistics and IRFs. We document that ASDs can alleviate these problems.

The next section explains the ASD procedure. Section 3 documents the ability of ASDs to detect and correct for misspecification using Monte Carlo experiments for a typical application. Section 4 discusses the results when our procedure is applied to the SW model on US data. Section 5 concludes.

2. Agnostic Structural Disturbances

We use a simple business cycle model to explain what ASDs are and how they can be used for building theoretical models that one wants to bring to the data. Appendix A, provides a general formulation.

2.1. Model

Agents' choices for consumption, C_t, investment, I_t, and capital, K_t are the outcomes of the following maximization problem:

$$\max_{\{C_{t+j},I_{t+j},K_{t+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \frac{C_{t+j}^{1-\gamma}-1}{1-\gamma} \tag{1}$$

s.t.

$$e^{\varepsilon_{a,t}} K_{t-1}^{\alpha} = C_t + I_t + e^{\varepsilon_{g,t}}\overline{G}, \tag{2}$$

$$K_t = (1-\delta)K_{t-1} + I_t e^{\varepsilon_{i,t}}. \tag{3}$$

This model contains three exogenous random variables. Using the terminology of Chari et al. (2007), these are an efficiency wedge, $\varepsilon_{a,t}$, an investment wedge, $\varepsilon_{i,t}$, and a government consumption wedge, $\varepsilon_{g,t}$. Consistent with the literature, the variables are generated by the following stochastic process:

$$\varepsilon_{m,t} = \rho_m \varepsilon_{m,t-1} + \sigma_m \eta_{m,t}, m \in \{a, i, g\}, \tag{4}$$

$$\mathbb{E}_t[\eta_{m,t+1}] = 0, \mathbb{E}_t[\eta^2_{m,t+1}] = 1, \text{and } \mathbb{E}_t[\eta_{m,t+1}\eta_{m^*,t+1}] = 0 \text{ for } m \neq m^*. \tag{5}$$

This economy is represented with the following set of linearized first-order conditions:[1]

$$\mathbb{E}_t[c_{t+1} - c_t] = \frac{1-\beta(1-\delta)}{\gamma}(\rho_a \varepsilon_{a,t} + (\alpha - 1)k_t) + \frac{1-\beta(1-\delta)\rho_i}{\gamma}\varepsilon_{i,t}, \tag{6a}$$

$$\overline{Y}(\varepsilon_{a,t} + \alpha k_{t-1}) = \overline{I}\; i_t + \overline{C}\, c_t + \overline{G}\, \varepsilon_{g,t}, \tag{6b}$$

$$k_t = (1-\delta)k_{t-1} + \frac{\overline{I}}{\overline{K}} i_t + \frac{\overline{I}}{\overline{K}}\varepsilon_{i,t}, \tag{6c}$$

where lower case letters denote variables expressed as a percentage difference from their steady state values and $\overline{X}$ indicates the steady state value of variable X_t.

The random disturbances can be interpreted literally as regular exogenous structural disturbances affecting the economy. As illustrated in Chari et al. (2007), however, these wedges can also be seen as manifestations of frictions in more elaborate models or as the part that is not modeled explicitly.[2] Although they are somewhat general, these three wedges *do* impose restrictions on the model and they differ from each other exactly because of these restrictions. First, none of the wedges appear in all equations, which is typical. Second, the model imposes cross-equation restrictions that depend on the structural parameter values of the model.[3]

[1]Throughout this paper, we focus on linearized systems and treat those as the true data generating process. We do this because most structural empirical macroeconomic models are based on such systems. In principle, one could include ASDs in nonlinear systems as well.

[2]Moreover, a wedge can be given different interpretations. For example, $\varepsilon_{g,t}$ could be a fixed cost to production or it could be government spending that agents do not value.

[3]Inoue et al. (2015) provide a formal analysis for using wedges to detect and identify misspecification. Their wedges also only appear in a limited set of equations and, thus, also do impose parameter restrictions.

The three policy functions for this model can be expressed as follows.

$$c_t = A_c(\Psi)k_{t-1} + B_{c,a}(\Psi)\varepsilon_{a,t} + B_{c,i}(\Psi)\varepsilon_{i,t} + B_{c,g}(\Psi)\varepsilon_{g,t}, \tag{7a}$$

$$i_t = A_i(\Psi)k_{t-1} + B_{i,a}(\Psi)\varepsilon_{a,t} + B_{i,i}(\Psi)\varepsilon_{i,t} + B_{i,g}(\Psi)\varepsilon_{g,t}, \tag{7b}$$

$$k_t = A_k(\Psi)k_{t-1} + B_{k,a}(\Psi)\varepsilon_{a,t} + B_{k,i}(\Psi)\varepsilon_{i,t} + B_{k,g}(\Psi)\varepsilon_{g,t}, \tag{7c}$$

where Ψ is a vector containing the structural parameters. This system also makes clear that wedges impose cross-equation restrictions. The $A_j(\Psi)$ and $B_{j,m}(\Psi)$ coefficients are nonlinear functions of the structural parameters, Ψ.[4] In linear frameworks, disturbances only differ in how they affect the economy on impact. After impact they propagate through the economy in the same way, as described by the A_js.

Possible misspecification Misspecification occurs in many different forms. One could miss a particular disturbance or include one that should not be included. Another possibility is that a structural disturbance is not incorporated correctly in all model equations. This is more likely to occur in larger models. However, misspecification is a also possible in the model at hand which has just three equations. For example, the government expenditure disturbance could very well affect the utility of the agent and/or the production function. Also, the investment disturbance may affect the depreciation rate.[5] Another possible misspecification is that, contrary to common practice, the structural disturbances are correlated with each other. Using a New Keynesian business cycle model, Cúrdia and Reis (2012) document that structural disturbances are correlated and ignoring this correlation leads to wrong inference.

2.2. Introducing Agnostic Structural Disturbances

ASDs can replace regular structural disturbances or they can be added to the existing set. Adding structural disturbances to model equations is incredibly simple: Each ASD is

[4] See Campbell (1998) for the derivation and discussion of such policy functions.

[5] In Section 4, we provide empirical evidence in support for this possibility.

added to *each* equation with a reduced form coefficient. When we add two ASDs, denoted $\widetilde{\varepsilon}_{A,t}$ and $\widetilde{\varepsilon}_{B,t}$, to the model of this section, then we get[6]

$$\mathbb{E}_t[c_{t+1} - c_t] = \frac{1-\beta(1-\delta)}{\gamma}(\alpha-1)k_t + [\widetilde{\Upsilon}_{1,A}\widetilde{\Upsilon}_{1,B}][\widetilde{\varepsilon}_{A,t}\widetilde{\varepsilon}_{B,t}]', \tag{8a}$$

$$\overline{Y}\alpha k_{t-1} = \overline{I}i_t + \overline{C}c_t + [\widetilde{\Upsilon}_{2,A}\widetilde{\Upsilon}_{2,B}][\widetilde{\varepsilon}_{A,t}\widetilde{\varepsilon}_{B,t}]', \tag{8b}$$

$$k_t = (1-\delta)k_{t-1} + \frac{\overline{I}}{\overline{K}}i_t + [\widetilde{\Upsilon}_{3,A}\widetilde{\Upsilon}_{3,B}][\widetilde{\varepsilon}_{A,t}\widetilde{\varepsilon}_{B,t}]', \tag{8c}$$

$$[\widetilde{\varepsilon}_{A,t}, \widetilde{\varepsilon}_{B,t}]' = \widetilde{\varepsilon}_t = \mathrm{P}\widetilde{\varepsilon}_{t-1} + \widetilde{\eta}_t. \tag{8d}$$

Each ASD is allowed to enter each equation without any restrictions. Moreover, they enter the system in a symmetric manner. A priori, there is, thus, no difference between the different ASDs. It is not restrictive to exclude future realizations of the ASDs from the equations. What matters is the expectation of these variables and this is captured by the current-period values as long as the ASDs are first-order Markov processes.

The vector $\widetilde{\eta}_t$ contains the ASD innovations. Their standard deviations can be normalized to 1, since the $\widetilde{\Upsilon}$s are reduced-form coefficients. ASD innovations are assumed to be uncorrelated, but the disturbances can be correlated because P does not have to be a diagonal matrix.[7] Thus, the vector with the ASD innovations, $\widetilde{\eta}_t$, satisfies

$$\mathbb{E}_t[\widetilde{\eta}_{t+1}] = 0 \text{ and } \mathbb{E}_t[\widetilde{\eta}_{t+1}\widetilde{\eta}'_{t+1}] = I_2. \tag{9}$$

The policy functions for this model with two ASDs can be expressed as follows.

$$c_t = A_c(\Psi)k_{t-1} + \widetilde{B}_{c,A}(\Psi)\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{c,B}(\Psi)\widetilde{\varepsilon}_{B,t}, \tag{10a}$$

$$i_t = A_i(\Psi)k_{t-1} + \widetilde{B}_{i,A}(\Psi)\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{i,B}(\Psi)\widetilde{\varepsilon}_{B,t}, \tag{10b}$$

$$k_t = A_k(\Psi)k_{t-1} + \widetilde{B}_{k,A}(\Psi)\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{k,B}(\Psi)\widetilde{\varepsilon}_{B,t}, \tag{10c}$$

[6] We have left out the three regular structural disturbances to keep the equations concise.

[7] Correlated innovations can be described by a combination of uncorrelated innovations. Such a setup is fine for agnostic disturbances.

where $A_c(\Psi)$ has the standard solution which does not depend on whether the disturbances are regular or ASDs. The coefficients in these equations are equal to

$$\widetilde{B}_{c,m} = \frac{\widetilde{\Upsilon}_{1,m} + (\Lambda - A_c(\Psi))\left(\widetilde{\Upsilon}_{3,m} - \frac{\widetilde{\Upsilon}_{2,m}}{\overline{K}}\right)}{(\Lambda - A_c(\Psi))\frac{\overline{C}}{\overline{K}} + \rho - 1}, \tag{11a}$$

$$\widetilde{B}_{i,,m} = -\frac{\overline{C}\widetilde{B}_{c,,m} + \widetilde{\Upsilon}_{2,m}}{\overline{I}}, \tag{11b}$$

$$\widetilde{B}_{k,m} = \frac{\overline{I}}{\overline{K}}\widetilde{B}_{i,m} + \widetilde{\Upsilon}_{3,m}, \tag{11c}$$

$$\Lambda = \frac{1 - \beta(1 - \delta)}{\gamma}(\alpha - 1). \tag{11d}$$

The expressions for the $\widetilde{B}_{j,m}(\Psi)$ coefficients illustrate the structural nature of ASDs because they depend both on the reduced-form $\widetilde{\Upsilon}$ coefficients and the structural parameters of the model, Ψ.

Although the $\widetilde{B}_{j,m}(\Psi)$ coefficients depend on Ψ, their values are fully unrestricted. That is, $\widetilde{B}_{c,m}(\Psi)$, $\widetilde{B}_{i,m}(\Psi)$, and $\widetilde{B}_{k,m}(\Psi)$ can take on any set of values by appropriate choice of $\widetilde{\Upsilon}_{1,m}(\Psi)$, $\widetilde{\Upsilon}_{2,m}(\Psi)$, and $\widetilde{\Upsilon}_{3,m}(\Psi)$. Since the $\widetilde{B}_{j,m}(\Psi)$ coefficients are unrestricted, an alternative way to implement ASDs is to add them directly to the policy functions with reduced-form coefficients, that is

$$c_t = A_c(\Psi)k_{t-1} + \widetilde{B}_{c,A}\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{c,B}\widetilde{\varepsilon}_{B,t}, \tag{12a}$$

$$i_t = A_i(\Psi)k_{t-1} + \widetilde{B}_{i,A}\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{i,B}\widetilde{\varepsilon}_{B,t}, \tag{12b}$$

$$k_t = A_k(\Psi)k_{t-1} + \widetilde{B}_{k,A}\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{k,B}\widetilde{\varepsilon}_{B,t}, \tag{12c}$$

This illustrates that the ASD procedure adds to the policy functions an unobserved components block. Describing time-series fully or partly with unobserved components has a rich history in macroeconomics.[8] This paper does more than that. Section 4 illustrates how ASDs can be used as a formal test of the correct specification of regular structural disturbances and how ASDs can be used to improve upon the specification of structural disturbances.

[8]See, for example, Stock and Watson (1999).

The data provide information about the coefficients of the unobserved components block, i.e., the $\widetilde{B}$ coefficients. However, to give an economically meaningful interpretation of the ASDs the $\widetilde{\Upsilon}$ coefficients are important. The link between the $\widetilde{B}$ and the $\widetilde{\Upsilon}$ coefficients will be discussed in more detail in Section 2.4.

2.3. ASDs and misspecification

ASDs can detect different types of misspecification. The procedure will indicate an additional structural disturbance is needed if *adding* an ASD improves model fit with the proper adjustment for the additional parameters introduced by the ASD. If *replacing* a regular structural disturbance by an ASD leads to improved model fit (adjusted for the number of parameters), then this indicates that the regular structural disturbance in question either needs to be modified or should not play a role in the empirical model.

Cúrdia and Reis (2012) test whether regular structural disturbances are *dynamically* correlated, that is, the innovations are orthogonal but lagged values of disturbances can affect current values of other disturbances. They find empirical evidence for such dynamic correlation.[9] ASDs can represent the role of correlated disturbances even if P is diagonal, which means that the ASDs are uncorrelated since the innovations of ASDs are assumed to be orthogonal. For example, suppose that both the efficiency wedge, $\varepsilon_{a,t}$ and the investment wedge, $\varepsilon_{i,t}$ are driven by a common component and an idiosyncratic component. Then an empirical model with three ASDs can capture the role of the three different random components. One vector of $\widetilde{\Upsilon}$ coefficients would capture the effect of the common component on the equations which would combine the effects of the $\varepsilon_{a,t}$ and the $\varepsilon_{i,t}$ disturbance. The other two $\widetilde{\Upsilon}$ vectors would capture the effects of the idiosyncratic components which would be the separate effect of the wedges.

ASDs can also capture measurement error. The ASD system as specified in Equation (8)

[9] Galizia (2015) shows that correlated estimates of the *innovations* of structural disturbances can be a sign of model misspecification. The paper demonstrates that the cross-correlations distorts the estimated variance decomposition of the model and proposes a method to mitigate this problem. As we show below, ASDs also help with getting a lower cross-correlation between the estimated innovations.

can capture measurement error in the control variables c_t and i_t.[10] To correctly represent measurement error in the state variable, k_t one would have to add lagged ASD values to the system.[11] Although ASDs are general enough to encompass measurement error, typical ASDs differ in a fundamental way from measurement error. In general, ASDs are structural disturbances and propagate through the system like regular structural disturbances, that is, according to the $A(\Psi)$ coefficients. Measurement error does not.[12]

ASDs are designed to deal with misspecification of structural disturbances that would distort the $B(\Psi)$ coefficients. Is the ASD procedure also able to deal with misspecification that affects the $A(\Psi)$ coefficients? Suppose one compares an empirical model with only one ASD with one that contains one regular structural disturbance and this disturbance is correctly specified. Moreover, both use $\widehat{A}(\Psi)$ which differs from the true $A(\Psi)$. The ASD specification can still fully represent the correct policy function as long as there is a $\widehat{\Psi}$ such that $\widehat{A}(\widehat{\Psi}) = A(\Psi)$. The specification with the regular structural disturbance faces a dilemma. With $\widehat{\Psi}$ it gets the A coefficient right, but the B coefficient wrong because it is improbable that $B(\widehat{\Psi}) = B(\Psi)$. If it chooses the correct value for Ψ then it gets B right but A wrong. The flexibility of the ASD procedure makes it more likely it gets the policy function coefficients right, not only in terms of the $\widetilde{B}$, but also in terms of the A coefficients. However, the example shows that this may come at the cost of larger distortions in estimates of Ψ if the ASD replaces a correctly specified regular structural disturbance.

Although our procedure can potentially alleviate misspecification of the $A(\Psi)$ matrix, we think of our procedure as a first step to understand where the model needs improvement not as a complete model evaluation.

[10] To see this simply replace c_t with $c_{\text{obs},t} + \varepsilon_t$ in Equation (6). After taking expectations one is left with just the current-value of the measurement error term, ε_t.

[11] Adding lagged values of the ASDs will increase the types of misspecification ASDs can detect. For example, this richer ASD specification could detect whether the period-t value of the productivity disturbance is known in period t, as is commonly assumed, or is known in period $t-1$, that is, when there are "news" shocks.

[12] When $\widetilde{\varepsilon}_{A,t}$ picks up measurement error in c_t or i_t, then the $\widetilde{B}_{k,A}$ coefficient associated with $\widetilde{\varepsilon}_{A,t}$ would be equal to zero. When an ASD in the enhanced system with lagged ASDs picks up measurement error in k_t, then there is also a set of restrictions such that the $A(\Psi)$ coefficients do not matter for the propagation of $\widetilde{\varepsilon}_{A,t}$.

2.4. Identification

The data provide information about the A and the B coefficients. To understand whether the A and the B coefficients can be estimated it is useful to think of the model variables as MA processes. With M ASDs, c_t is a sum of M MA processes. The parameters of each MA process depend on: $A_c(\Psi)$, $A_k(\Psi)$, $\widetilde{B}_{c,m}$, and $\widetilde{B}_{k,m}, m \in \{1, \cdots, M\}$. Thus if one uses just c_t as an observable, then one can estimate these A and $\widetilde{B}$ coefficients, but one would not be able to estimate the ASD coefficient in the investment policy function, $\widetilde{B}_{i,m}$. If one replaces regular structural disturbances with ASDs, then it may be harder to identify Ψ. That turns out not to be an issue in our empirical application presented in Section 4.

The As and $\widetilde{B}$s determine the policy functions and moment properties. These may provide some information on the nature of the ASD. The $\widetilde{\Upsilon}$s indicate how ASD enters each and every equation. In the empirical application in Section 4, we find that knowing the $\widetilde{\Upsilon}$s is especially useful for interpreting the different ASDs. So the question arises whether knowing the $\widetilde{B}$s is enough to determine the $\widetilde{\Upsilon}$s. Equation (11) makes clear that knowing $\widetilde{B}_{c,m}$, $\widetilde{B}_{i,m}$, and $\widetilde{B}_{k,m}$ is necessary but not sufficient to determine $\widetilde{\Upsilon}_{1,m}$, $\widetilde{\Upsilon}_{2,m}$, and $\widetilde{\Upsilon}_{3,m}$.[13] In addition, one would need certain combinations of the structural parameters.[14]

2.5. ASDs versus DSGE-VARs

Ireland (2004) and Del Negro et al. (2007) combine a DSGE model with a reduced-form VAR that contains the observables. There are several key differences between these two approaches and ours.

The DSGE-VAR specification is best compared with the system given in Equation (12) which adds ASDs to the policy functions. However, an advantage of the ASD procedure is that one can also obtain the specification given in Equation (8) that determines how the disturbances affect model equations. This knowledge is helpful in interpreting the nature of

[13]To estimate all three $\widetilde{B}$ coefficients one would need data on both consumption and investment.

[14]For example, the following expression would need to be identified: $\frac{(\Lambda - A_c(\Psi))}{(\Lambda - A_c(\Psi))\frac{C}{K}+\rho-1}$.

the ASDs as shown in Section 4. This cannot be done with the DSGE-VAR approach.

The ASD approach focuses on a particular type of misspecification, which allows it to use aspects of the model that are assumed to be not affected by the misspecification, namely $A(\Psi)$. The DSGE-VAR approach is more ambitious and also directly considers misspecification of $A(\Psi)$. Introducing a VAR into the empirical model means that the number of disturbances necessarily increases by a number equal to the number of variables in the VAR. Moreover, adding a VAR introduces many more parameters unless the number of observables is small. By contrast, our procedure allows for a more parsimonious approach and could consist of adding just one new disturbance or replacing one regular structural disturbance with an agnostic structural disturbance.

These differences imply that our approach is more efficient in terms of the number of parameters that it has to estimate.[15] The price of parsimony is that our procedure is not designed to detect misspecification unrelated to structural disturbances, that is, misspecification associated with restrictions imposed by $A(\Psi)$. However, as discussed above the flexibility of our procedure may still alleviate misspecification of $A(\Psi)$. The DSGE-VAR approach explicitly allows misspecification in $A(\Psi)$. However, Chari et al. (2008) point out that the VAR with a finite number of lags that does not contain *all* the model's state variables is likely to be misspecified. This means that the DSGE-VAR approach cannot deal with all possible misspecifications either.

Another difference emerges as the sample size goes to infinity. With the DSGE-VAR approach one has two "competing" empirical specifications, a DSGE model and a VAR. Since every DSGE suffers from at least some minor misspecification, one can expect the VAR to fully take over as the sample size goes to infinity. If that happens, then one is left with a reduced-form model. This will never happen with our approach, since the propagation of state variables will always be determined by $A(\Psi)$.

[15]For example, for the popular DSGE model of Smets and Wouters (2007) with 7 observables, a VAR with 4 lags would mean estimating 204 additional coefficients. As discussed in Section 4, the implementation of our procedure for this model means estimating twelve more parameters.

3. Small-Sample Monte Carlo experiments

In this section, we use two small-sample Monte Carlo experiments to demonstrate that ASDs can be used to detect and correct for misspecification in a typical empirical application. As a byproduct, it is shown that the consequences of minor misspecifications in modeling the regular structural disturbances can lead to large distortions in terms of parameter estimates deviating from their true values. Each experiment consists of 1,000 replications. Additional details and results are discussed in Appendix C.

3.1. True model and empirical specifications.

We use the New Keynesian model of Smets and Wouters (2007), the workhorse model of empirical business cycle analysis, to generate the data for each Monte Carlo replication.

The misspecification of the empirical model. The original SW model has seven exogenous random variables. Those are a TFP disturbance, $\varepsilon_{a,t}$, a risk-premium disturbance, $\varepsilon_{b,t}$, a government spending disturbance, $\varepsilon_{g,t}$, an investment-specific disturbance, $\varepsilon_{i,t}$, a monetary policy disturbance, $\varepsilon_{r,t}$, a price mark-up disturbance, $\varepsilon_{p,t}$, and a wage mark-up disturbance, $\varepsilon_{w,t}$. We leave out one of these seven disturbances when generating data for our misspecification experiments. The empirical specification also leaves out one disturbance, *but* not the right one. Every other aspect of the empirical model is correctly specified, including functional forms, specification of the processes for the exogenous random variables, and the values of the parameters that are not estimated.

These are computationally expensive exercises and we only discuss two of the possible forty-two combinations in detail in this section. In the first Monte Carlo experiment, the true *dgp* does not include the monetary policy disturbance, but the empirical model leaves out the investment disturbance instead. In the second disturbance, the empirical model also leaves out the investment disturbance, but it differs from the first in that the true *dgp* does not include the TFP disturbance. In Appendix D, we abstract from small-sampling noise and discuss all forty-two experiments in detail. The appendix also shows that distortions in parameter estimates carry over to implied model properties and explains why we chose these two experiments for this section's Monte Carlo experiments.

Is this a "minor" misspecification? When generating the data, we adjust the standard deviation of the disturbance that is incorrectly excluded from the empirical specification to ensure that it is responsible for at most 10% of the volatility for *any* of the six observables used in the estimation. This reduces the quantitative importance of the misspecification.

One could argue that a misspecification is only minor if one would not detect it in a typical data set using some model selection criterion such as the marginal likelihood. This is a very strict requirement. Comparing a misspecified model with the true one requires that researchers are aware of the correct specification and test their empirical model against it. Since structural disturbances can enter models in many different ways, researchers may not consider the correct one even if they consider several alternatives. Nevertheless, we implement this test adopting the Bayesian estimation methodology used in Smets and Wouters (2007) with the same priors. Using the marginal data density, the misspecified specification is preferred over the true specification in 17% and 47% of the generated samples for the first and the second Monte Carlo experiment, respectively.

Is this a likely misspecification? We believe that this type of misspecification is likely to be important in practice even if one includes a large set of structural disturbances. The first reason is that having a large set does not necessarily imply one includes all the true disturbances. Moreover, one does not only need to include all true disturbances, *each* disturbance has to enter *each* model equation correctly. For example, a TFP disturbance is typically modeled as a labor-augmenting productivity shock, but productivity changes could affect the production function differently. Moreover, TFP may also affect other aspects of the production process such as the depreciation rate. Moreover, one could argue, that this misspecification is not that likely for the analysis in Smets and Wouters (2007), since SW was preceded by years of empirical analysis by many authors. In Section 4, however, we document that we clearly reject the null that two of the included structural disturbances are correctly specified.

Observables and sample size. The set of observables used in SW consists of employment, the federal funds rate, the inflation rate, GDP, consumption, investment, and the real wage rate. We exclude the real wage rate so we have the same number of observables as structural disturbances which is consistent with the empirical exercise in SW. We use a sample of typical length, namely 156, which is the same as the number of observations used to estimate the model in Smets and Wouters (2007).

Estimation procedure. DSGE models are typically estimated with Bayesian techniques, which means that the estimation outcome is a weighted combination of the prior and the empirical likelihood. Misspecification of the empirical model affects the latter. Observed data – and thus misspecification of the likelihood – matter less for posterior estimates with a tight prior. The quality of the estimates will then depend on the quality of the prior. This paper focuses on the question how misspecification affects what the observed data imply for parameter estimates. Thus, we focus on the likelihood and use Maximum Likelihood (ML) estimation. We do impose bounds on the range of parameters considered which alleviates the complexity of the optimization problem.

Priors on the standard deviation of structural disturbances typically do not allow for point mass at zero. Ferroni et al. (2015) point out that this biases the results towards a positive role of all structural disturbances. This is not an issue for us, since we use ML estimation. In fact, estimated standard deviations of disturbances that are part of the empirical model but *not* part of the true *dgp* turn out to be often close to zero. Parameter values of the true data generating process are set equal to those of the SW posterior mode. The list of parameters estimated and their interpretation is given in Table 1.

3.2. Evaluating the performance of the ASD approach

In this section, we discuss the results of our Monte Carlo experiments. The outcomes for three different empirical models are compared. The first empirical specification correctly models all regular structural disturbances as in Smets and Wouters (2007). This approach is denoted SW. The second empirical model excludes one regular structural disturbance that is part of and includes one regular structural disturbance that is not part of the true *dgp*. The third empirical model also excludes one of the regular structural disturbances, but replaces it with an ASD. This ASD empirical model is of a more reduced-form nature than the SW specification, but it is not misspecified. That is, there are values of the reduced-form parameters such that it matches the true model.

Section 3.2.1 discusses the results when ASDs are used to detect misspecification both when the empirical model is indeed misspecified and when it is not. Section 3.2.2 discusses the ability of ASDs to correct for misspecification.

3.2.1. Using ASDs to detect misspecification

A good test for misspecification has power to reject a misspecified model and rejects a correctly specified model at the chosen significance level. This section documents that ASDs are capable of doing both.

Case I: The empirical model is not correctly specified. To evaluate whether the ASD procedure can detect misspecification, we use a Likelihood Ratio (LR) test that compares the likelihood of the agnostic empirical specification to the likelihood of the misspecified empirical model. The number of degrees of freedom is equal to ten, since the agnostic specification has ten more parameters.[16] With this procedure, the ASD procedure rejects the misspecified model in all Monte Carlo replications in both experiments. The procedure is, thus, quite powerful in detecting misspecification. The power of the test would decrease if one would use a Bayesian approach, since the common prior would make the posterior of the empirical model with an ASD and the misspecified empirical model more similar and less dependent on the data.

[16] We use the formulation of our procedure that adds ASDs directly to the policy functions. This formulation introduces the smallest possible number of additional parameters.

Case II: The empirical model is correctly specified. For the first Monte Carlo experiment, we find that the rejection rate is 21.5% at the 10%-level and 12% at the 5%-level. For the second experiment, these two numbers are 20.9% and 12.6%. The standard error for an estimated fraction is given by $\widehat{f}(1-\widehat{f})/\sqrt{1000}$, so these differences are significantly different from their theoretical counterpart. Although these small-sample results do not coincide precisely with the theoretical predictions based on large-sample theory, the distortions are not unreasonable. In Appendix C, we document that the histograms of estimated χ^2 statistics are reasonably close to the theoretical (large-sample) χ^2 distribution, but – as indicated by the numbers above – have a slightly fatter upper tail.

3.2.2. Using ASDs to correct for misspecification

The discussion above made clear that the ASD procedure does very well in terms of detecting misspecified models and reasonably well in not rejecting correctly specified models in small samples. In this subsection, we document that the estimates of the structural parameters obtained with the agnostic procedure are much closer to the true values than those obtained with the misspecified empirical model. In fact, they are very similar to those obtained with the correctly specified empirical model with all structural disturbances fully modeled.

Table 2 reports the average absolute error of the parameter estimates relative to the true value for the three different empirical models across Monte Carlo experiments. Parameter estimates obtained with the misspecified structural model are substantially worse than those obtained with the correctly specified model. The average of the errors for the misspecified model is more than twice as large as the one for the -fully-specified SW model for several parameters and for both experiments.[17] For the misspecified model, the average errors are typically better for the second than for the first experiment. However, that is not true for all

[17] Particular problematic is the standard deviation of the TFP disturbance in the first Monte Carlo experiment for which the average error is almost nine time as large as the one for the correct empirical model. Consistent with the broader investigation of Appendix D, this disturbance often takes over the role of the wrongly excluded structural disturbance.

parameters. For example, the average error for σ_c is substantially higher in the second experiment, whereas there is only a modest increase for the correctly specified model. Appendix D, which discusses the consequences of misspecification for all forty-two experiments, shows that the substantial distortions in parameter estimates reported here are not atypical and also distort implied model properties such as business cycle moments and IRFs.

For the first Monte Carlo experiment, the average errors for the agnostic setup and the SW specification are very similar. Although only slightly, the average error is actually lower for the agnostic specification for ten of the twenty-seven parameters. Note that the agnostic specification is not misspecified, but has a disadvantage relative to the SW specification since it uses a reduced-form approach and contains ten more parameters. Nevertheless, the efficiency loss turns out to be very minor.

For the second Monte Carlo experiment, the SW specification comes with some noticeable efficiency advantages for some parameter estimates. Nevertheless, estimates obtained with the ASD procedure are still much better than the one obtained with the misspecified model.

Figures 1 and 2 plot histograms characterizing the distribution of the parameter estimates across Monte Carlo replications for a selected set of parameters. Each panel reports the results for the fully-specified SW model (dark line and dots), the agnostic procedure (white bars), and the misspecified model (blue/dark bars). The figures document that the distributions of estimates obtained with the SW specification and the agnostic procedure are both qualitatively and quantitatively very similar. By contrast, the distribution of estimates obtained with the misspecified empirical model can be vastly different. For example, Panel a of Figure 1 documents that the distribution of estimates of the capital share parameter, α, displays a strong downward bias when the misspecified empirical model is used. The associated mean is equal to 0.09, whereas the true value is equal to 0.19. The figure also documents that a large number of estimates are clustered at the imposed lower bound. That is, by imposing bounds we limited the distortions due to misspecification. For α, the leftward shift is so large, that there is little overlap between the distribution of the estimates based on the misspecified model and the other two empirical models. Bunching at the lower or upper bound is more pervasive for the first experiment, but also observed for the second.

For the parameters considered in these figures, the distribution of estimates for the agnostic and the fully-specified SW specification are almost always centered around the true parameter value. In principle, there could be a small sample bias, since this is a complex nonlinear estimation problem. The full set of results, discussed in Appendix C, do indeed indicate that there is a bias for some parameters. In those cases, the bias is similar for the estimator based on the fully-specified specification and the agnostic one. An example of a parameter that is estimated with bias is the labor supply elasticity with respect to the real wage, σ_l. Its true value is equal to 1.92. In the first experiment, the average estimate across the Monte Carlo replications is equal to 1.84 for the SW and 1.71 for the agnostic specification. By contrast, the associated average estimate is equal to 0.27 for the misspecified model, which indicates a bias of a much larger magnitude.

4. Are the SW disturbances the right ones for US data?

In this section, we first apply the ASD procedure to test the restrictions imposed by the SW structural disturbances with the US postwar data used by SW. We document that the restrictions imposed by the risk premium and the investment-specific technology disturbance are rejected by the ASD procedure. Next, we use model selection procedures to determine the number of ASDs to include and to construct a more concise specification that excludes the agnostic disturbances from some model equations. To conclude, we interpret the nature of the agnostic structural disturbances by examining the sign and magnitude of their associated coefficients in model equations and their IRFs. Appendix E provides more detailed information on our empirical analysis and additional results.

4.1. Testing the Smets-Wouters disturbance restrictions

Since SW use a Bayesian estimation procedure, we do the same. Implementing the ASD procedure only requires a minor modification of the Dynare program that estimates the model for the original SW specification. Replacing an SW regular structural disturbance with an ASD introduces a 13×1 $\widetilde{\Upsilon}$ vector but only twelve additional parameters to estimate, since the standard deviation of the ASD innovation is normalized to 1.

Suppose an original SW disturbance enters the j^{th} equation with coefficient $\widetilde{\Upsilon}_j(\Psi)$. When it is replaced with an ASD, then we set the prior for the ASD coefficient in the j^{th} equation to a Normal with a mean equal to $\widetilde{\Upsilon}_j(\Psi)$ with Ψ evaluated at SW prior means. By centering the priors of the agnostic coefficients around the SW restrictions, we favor the SW specification. However, the means of these priors hardly matter and our results are robust to setting the prior mean equal to zero for all coefficients. The standard deviations of the prior distributions for the $\widetilde{\Upsilon}$ coefficients are set equal to 0.5. This implies very uninformed priors, since the model is linear in log variables. As a robustness check we also consider a standard deviation equal to 0.1 and we find very similar results.

The specification that replaces a regular disturbance with an agnostic one encompasses the original specification which gives it an advantage in terms of achieving a better fit. The additional parameters, however, act as a penalty term in the marginal data density. Table 3 reports the marginal data densities for the original SW specification and for specifications in which the indicated regular structural disturbances is replaced by an ASD. Overall, these outcomes are quite supportive of the original SW specification as the SW restrictions are preferred for five of the seven structural disturbances. But the results for the risk-premium and the investment specific disturbance indicate that improvement is possible.

4.2. Obtaining our preferred model with ASDs

These results do not necessarily imply that we should exclude the structural risk-premium and investment disturbance. After all, it is possible that a model that includes agnostic disturbances *as well as* these two SW structural disturbances has an even higher marginal data density. Moreover, ASDs add quite a few extra parameters which may make interpretation more difficult. The next step of the ASD procedure is to use a model selection procedure.

There are different model selection procedures one can use to obtain a preferred specification. Our procedure is described in detail in Appendix E.2. The chosen model is one that excludes the SW risk premium as well as the SW investment disturbance, it includes three

ASDs, and imposes several zero restrictions on the ASD coefficients.

4.3. Giving the ASDs an economic interpretation

ASDs are agnostic by nature. The model selection procedure also does not use any economic reasoning. Here we will show how the estimation results, such as parameter estimates of ASD coefficients and IRFs, can be used to give a meaningful interpretation to the ASDs. We will argue that one of the three selected ASDs can be interpreted as an investment-specific disturbance, but with some quite striking differences from the regular one used in the literature and in SW. We will refer to this ASD as the agnostic "investment-modernization disturbance." The second ASD has features in common with the SW risk-premium disturbance and with a preference disturbance, but is different from both. We will refer to this ASD as the agnostic "Euler disturbance." The role of the third ASD is quantitatively less important than the other two. It mainly affects wage growth and is associated with a more efficient use of capital. We will refer to this ASD as the "capital-efficiency wage mark-up disturbance." By assigning names to agnostic disturbances, we may open ourselves to criticism. Our main reason for assigning these names is that we want to make clear that agnostic disturbances are in principle theory-free, *and yet* allow the researcher to go one step further, towards giving an economic interpretation to them.

4.3.1. The agnostic investment-modernization disturbance, $\widetilde{\varepsilon}_{B,t}$

In the SW model, the investment-specific technology disturbance shows up in the investment Euler equation and in the capital accumulation equation. One of our agnostic disturbances, $\widetilde{\varepsilon}_{B,t}$, also shows up in these two equations.[18] The only other equation in which $\widetilde{\varepsilon}_{B,t}$ appears is the equation that relates capacity utilization to the rental rate of capital. These findings indicate that $\widetilde{\varepsilon}_{B,t}$ could be interpreted as an investment-specific productivity

[18] In our computer programs, the ASDs are referred to as agnA, agnB, and agnC. The interpretation for agnB is the most straightforward so we discuss this one first. We could have relabeled it as $\widetilde{\varepsilon}_{A,t}$, but chose not to do so to emphasize that labels for ASDs are arbitrary.

disturbance. Furthermore, as documented in Table 4, $\widetilde{\varepsilon}_{B,t}$, plays an important role for the volatility of investment. Specifically, it explains 70% of the volatility of investment growth compared to 82.1% for the investment-specific disturbance in the SW model. Interestingly, $\widetilde{\varepsilon}_{B,t}$ is not important for the volatility of capital. Specifically it only explains 2.37% of the volatility of the capital stock, whereas the SW investment disturbance explains 32.5%. Thus, if $\widetilde{\varepsilon}_{B,t}$ is an investment-specific disturbance, then it is not a typical one.

Figure 3 plots the IRFs of our agnostic disturbance and the SW investment-specific disturbance. This graph documents there are some remarkable differences. The SW investment disturbance generates a typical business cycle with key aggregates moving in the same direction. A positive agnostic investment disturbance also leads to a strong comovement between output and investment, but leads to a *reduction* in consumption and capital.[19] Also, whereas capacity utilization decreases in the SW model, our specification indicates an increase.

To understand these differences and to explain why we still think that $\widetilde{\varepsilon}_{B,t}$ is an investment-specific disturbance, we have to take a closer look at the relevant equations and how $\widetilde{\varepsilon}_{B,t}$ affects these equations differently than the SW investment specific disturbance, $\varepsilon_{i,t}$. The three relevant equations are the following:[20]

Smets-Wouters investment-specific disturbance, $\varepsilon_{i,t}$

$$\text{Investment Euler:} \quad i_t = i_1(\Psi)\, i_{t-1} + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \boldsymbol{\varepsilon_{i,t}}, \tag{13}$$

$$\text{Utilization:} \quad z_t = z_1(\Psi)\, r_t^k, \tag{14}$$

$$\text{Capital:} \quad k_t = k_1(\Psi)\, k_{t-1} + (1 - k_1(\Psi))\, i_t + \boldsymbol{k_2(\Psi)\, \varepsilon_{i,t}}, \quad \boldsymbol{k_2(\Psi) > 0}. \tag{15}$$

[19] Justiano et al. (2010) also report a negative consumption response to an investment disturbance, but only for the first five periods. As discussed in Ascari et al. (2016), most models would predict a countercyclical consumption response to an investment disturbance. The SW model overturns this property due to a sufficiently high degree of price and wage stickiness. Our agnostic approach implies similar estimates for price and wage stickiness, but still indicates that the data prefer a countercyclical consumption response.

[20] The subscripts of the coefficients of the agnostic disturbance refer to the SW equation number. For example, $\widetilde{\Upsilon}_{3,B}\widetilde{\varepsilon}_{B,t}$ is the term added to Equation (3) of SW. i_t is the investment level, r_t^k the rental rate of capital, z_t the utilization rate, $\varepsilon_{i,t}$ the SW investment-specific investment disturbance, and Ψ is the vector with structural parameters.

Agnostic investment-modernization disturbance, $\widetilde{\varepsilon}_{B,t}$

$$\text{Investment Euler:} \quad i_t = i_1(\Psi)\, i_{t-1} + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \boldsymbol{\widetilde{\Upsilon}_{3,B}\widetilde{\varepsilon}_{B,t}}, \ \boldsymbol{\widetilde{\Upsilon}_{3,B} > 0}, \tag{16}$$

$$\text{Utilization:} \quad z_t = z_1(\Psi)\, r_t^k + \boldsymbol{\widetilde{\Upsilon}_{7,B}\widetilde{\varepsilon}_{B,t}}, \quad \boldsymbol{\widetilde{\Upsilon}_{7,B} < 0}, \tag{17}$$

$$\text{Capital:} \quad k_t = k_1(\Psi)\, k_{t-1} + (1 - k_1(\Psi))\, i_t + \boldsymbol{\widetilde{\Upsilon}_{8,B}\widetilde{\varepsilon}_{B,t}}, \quad \boldsymbol{\widetilde{\Upsilon}_{8,B} < 0}. \tag{18}$$

The reason for the striking differences between the IRFs of our ASD and the SW investment disturbance is that our unrestricted approach lets the agnostic investment-specific disturbance appear in the capital accumulation equation *without* restrictions. That is, the sign of the coefficient of $\widetilde{\varepsilon}_{B,t}$, $\widetilde{\Upsilon}_{8,B}$, is unrestricted, but the coefficient of $\varepsilon_{i,t}$ in the SW specification, $k_2(\Psi)$ is restricted by the values of the structural parameters, Ψ. The outcome is that the posterior mean of $\widetilde{\Upsilon}_{8,B}$ has the *opposite* sign relative to $k_2(\Psi)$ and the 90% HPD does not include 0.

This means that a reduction in the cost of transforming current investment into capital goes together with increased depreciation of the existing capital stock in our specification. In the SW model, an investment-specific disturbance does not affect the economic viability of the existing capital stock. Our agnostic approach questions this assumption and suggests that the investment-specific productivity disturbance goes together with scrapping of older vintages. This is the reason why we refer to it as an agnostic investment-modernization disturbance.

In the SW model, capacity utilization is proportional to the rental rate and there are no shocks that can affect this relationship. An accelerated depreciation of the capital stock increases the rental rate, which in turn would induce an increase in the utilization rate. In our agnostic specification, this relationship is dampened somewhat, since a positive agnostic disturbance has a *direct* negative impact on capacity utilization, since it enters the capacity utilization with a negative coefficient. The overall effect is still an increase in capacity utilization. It seems plausible that scraping of old vintages goes together with higher utilization of the remaining capital stock.

4.3.2. The agnostic Euler disturbance, $\widetilde{\varepsilon}_{A,t}$

The agnostic disturbance $\widetilde{\varepsilon}_{A,t}$ appears in eight equations. The key equation is the Euler equation for bonds, because excluding the disturbance from this equation leads to by far the largest drop in the marginal data density. This suggests that it could have key characteristics in common with a preference or a risk-premium disturbance. This view is also supported by Table 4 which documents that $\widetilde{\varepsilon}_{A,t}$ is important for the same variables as the SW risk-premium disturbance. However, this agnostic disturbance also has some quite different characteristics from both. Therefore, we will adopt an alternative name and refer to it as the agnostic Euler disturbance. For the interpretation of $\widetilde{\varepsilon}_{A,t}$, it is important to understand the differences in impact of a regular preference and a regular (bond) risk-premium disturbance.

Difference between a preference and (bond) risk-premium disturbance. Smets and Wouters (2003) include a preference disturbance which affects current utility. This means it affects the marginal rate of substitution and, thus, *all* Euler equations. By contrast, Smets and Wouters (2007) include instead a (bond) risk premium that introduces a wedge between the policy rate and the required rate of return on bonds without affecting other Euler equations.[21] Both disturbances have a strong impact on current consumption when prices are sticky. A positive preference disturbance reduces the attractiveness of *all* types of saving including investment. A positive risk-premium disturbance only makes savings in bonds less attractive. That is, it induces a desire to substitute out of bonds and into investment, in addition to an increase in consumption. Thus, a preference disturbance leads to a negative comovement of consumption and investment, whereas a (bond) risk-premium disturbance leads to a positive comovement.

Also, a preference disturbance affects output in both the flexible-price and the sticky-price part of the model, whereas a risk-premium disturbance has no affect on key aggregates such as consumption and output in the flexible price part of the SW model.

[21] If a preference disturbance is added to the specification of Smets and Wouters (2007), then the marginal data density drops from -922.40 to -923.57 and the preference disturbance plays virtually no role.

Is $\widetilde{\varepsilon}_{A,t}$ a preference, a risk-premium, or another type of disturbance? Figure 4 plots the IRFs of the SW risk-premium and our agnostic disturbance. The figure documents that both generate a regular business cycle with positive comovement for output, consumption, investment, and hours. The positive comovement suggest that the agnostic disturbance is a bond risk-premium disturbance and not a preference disturbance. However, the agnostic disturbance has a strong impact on flexible-price output which is inconsistent with it being a (bond) risk-premium disturbance and consistent with it being a preference disturbance. Since this ASD differs from both a preference and a risk-premium disturbance, we come up with a new term, namely the Euler disturbance.

To better understand the nature of the agnostic Euler disturbance, we take a closer look at the equations in which $\widetilde{\varepsilon}_{A,t}$ enters. It appears in the aggregate budget constraint, the bond Euler equation, the investment Euler equation, the capital value equation, the utilization rate equation, the price mark-up equation, the rental rate of capital equation, and the Taylor rule. Although $\widetilde{\varepsilon}_{A,t}$ affects quite a few different aspects of the model, the interpretation is eased by the fact that its role is minor in most of the eight equations in the sense that allowing it to enter these equations only has a minor quantitative impact on the behavior of model variables or only affects the qualitative behavior of one or two variables without affecting the behavior of the key macroeconomic variables.

Specifically, to understand the role of $\widetilde{\varepsilon}_{A,t}$ on key macroeconomic aggregates we can restrict ourselves to the Taylor rule and the three model equations that are relevant for the savings/investment decisions, which are the bond Euler equation, the investment Euler equation, and the capital value equation. As in SW, we use the bond Euler equation to substitute the marginal rate of substitution out of the capital valuation equation. While the SW bond risk-premium disturbance, $\varepsilon_{b,t}$, does *not* appear in the original capital valuation equation, it does show up *after* this substitution has taken place. Moreover, it appears in these two equations with the exact same coefficient as the nominal interest rate for bonds, r_t. By contrast, *after* substituting out the marginal rate of substitution in the capital value equation, a preference disturbance would *no longer* appear in the capital valuation equation. The following set of equations documents how the SW risk-premium and our agnostic Euler disturbance enter these equations:[22]

Smets-Wouters risk premium, ε_t^b

$$\text{Bond Euler:} \quad c_t = c_1(\Psi)\, c_{t-1} + (1 - c_1(\Psi))\, \mathbb{E}_t[c_{t+1}] + c_2(\Psi)\,(l_t - \mathbb{E}_t[l_{t+1}])$$
$$-c_3(\Psi)\, \boldsymbol{(r_t - \mathbb{E}_t[\pi_{t+1}] + \varepsilon_{b,t})},\, c_3(\Psi) > 0, \tag{19}$$
$$\text{Inv. Euler:} \quad i_t = i_1(\Psi)\, i_{t-1} + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \varepsilon_{i,t}, \tag{20}$$
$$\text{Valuation:} \quad q_t = q_1 \mathbb{E}_t[q_{t+1}] + (1 - q_1)\mathbb{E}_t\left[r_{t+1}^k\right]$$
$$-\boldsymbol{(r_t - \mathbb{E}_t[\pi_{t+1}] + \varepsilon_{b,t})}, \tag{21}$$
$$\text{Policy rate:} \quad r_t = \rho r_{t-1} + (1 - \rho)\{r_\pi + r_Y(y_t - y_t^p)\}$$
$$+r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_{r,t}. \tag{22}$$

Agnostic Euler disturbance, $\widetilde{\varepsilon}_{A,t}$

$$\text{Bond Euler:} \quad c_t = c_1(\Psi)\, c_{t-1} + (1 - c_1(\Psi))\, \mathbb{E}_t[c_{t+1}] + c_2(\Psi)\,(l_t - \mathbb{E}_t[l_{t+1}])$$
$$-c_3(\Psi)\, \boldsymbol{(r_t - \mathbb{E}_t[\pi_{t+1}]) - \widetilde{\Upsilon}_{2,A}\widetilde{\varepsilon}_{A,t},\, \widetilde{\Upsilon}_{2,A} > 0}, \tag{23}$$
$$\text{Inv. Euler:} \quad i_t = i_1(\Psi)\, i_{t-1} + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \varepsilon_{i,t} - \boldsymbol{\widetilde{\Upsilon}_{3,A}\widetilde{\varepsilon}_{A,t},\, \widetilde{\Upsilon}_{3,A} > 0}, \tag{24}$$
$$\text{Valuation:} \quad q_t = q_1 \mathbb{E}_t[q_{t+1}] + (1 - q_1)\mathbb{E}_t\left[r_{t+1}^k\right]$$
$$-\boldsymbol{(r_t - \mathbb{E}_t[\pi_{t+1}]) - \widetilde{\Upsilon}_{4,A}\widetilde{\varepsilon}_{A,t},\, \widetilde{\Upsilon}_{4,A} > 0}, \tag{25}$$
$$\text{Policy rate:} \quad r_t = \rho r_{t-1} + (1 - \rho)\{r_\pi + r_Y(y_t - y_t^p)\}$$
$$+r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_{r,t} + \boldsymbol{\widetilde{\Upsilon}_{14,A}\widetilde{\varepsilon}_{A,t},\, \widetilde{\Upsilon}_{14,A} > 0}. \tag{26}$$

Our ASD appears in the bond Euler equation and the capital valuation equation and it shows up with the same sign as the SW risk-premium disturbance. This supports the view that our ASD is similar to a risk-premium disturbance. Nevertheless, one could argue that

[22]In these equations, c_t is consumption, l_t is hours worked, r_t is the nominal policy rate, π_t is the inflation rate, q_t is the price of capital, y_t is output, and y_t^p is output in the flexible-price economy. Also see the information given in footnote 20.

the ASD is a preference and not a bond risk-premium disturbance for the following reasons. Although $\widetilde{\Upsilon}_{4,A}$ has the right sign for a risk-premium coefficient, its magnitude, evaluated using the posterior mean, is way too small.[23] The 90% HPD interval of the coefficient of $\widetilde{\varepsilon}_{A,t}$ in the capital valuation equation, $\widetilde{\Upsilon}_{4,A}$, includes zero and setting the coefficient equal to zero has very little impact on model properties and virtually none on the marginal data density.

As pointed out above, a preference disturbance generates consumption and investment responses that move in opposite directions. Our ASD predicts responses in the same direction even if we impose that the ASD does not enter the capital valuation equation (after substituting out the MRS). The reason for the positive comovement is that our ASD also enters the investment Euler equation. The investment Euler equation is a dynamic equation, but its dynamic aspects are due solely to investment adjustment costs.[24] Our agnostic approach indicates that the structural disturbance that plays a key role in the bond Euler equation should also appear in the investment Euler equation. In fact, it is the first equation chosen in our specific-to-general model selection procedure.

What could this agnostic disturbance represent? A simple explanation is that it is a preference disturbance that is correlated with an investment-specific disturbance. This disturbance appears directly in the Taylor rule with a negative coefficient. This means that the central bank responds more aggressively to business cycle fluctuations induced by this Euler disturbance. Without this effect on the Taylor rule this disturbance would have a stronger impact on economic aggregates and inflation would no longer be procyclical.

4.3.3. The agnostic capital-efficiency wage mark-up disturbance, $\widetilde{\varepsilon}_{C,t}$

The third ASD chosen by our model selection criterion increases the total number of structural disturbances to eight, that is, one more than the number in the SW specification. Thus, this ASD cannot be interpreted as a replacement of a SW disturbance.

[23]If our ASD is a risk-premium disturbance, then $\widetilde{\Upsilon}_{4,A}/\widetilde{\Upsilon}_{2,A}$ should be equal to $1/c_3(\Psi)$, but using posterior means, we find that $\widetilde{\Upsilon}_{4,A}/\widetilde{\Upsilon}_{2,A} = 3.3$, whereas $1/c_3(\Psi) = 7.27$, substantially higher.

[24]Adjustment costs are zero in the steady state, which implies that neither a preference disturbance nor a risk-premium disturbance appear in a *linearized* investment Euler equation. A preference disturbance would appear in the original *nonlinear* equation.

This third ASD, $\widetilde{\varepsilon}_{C,t}$, appears in five equations and the most important one (in terms of impact on the marginal data density) is the wage-adjustment equation. It also shows up into three equations related to capital, namely the capital accumulation equation, the capital utilization equation, and the capital-valuation equation. Finally, it appears in the economy-wide budget constraint, although the impact on the latter is minor.

Given its impact on the wage equation, this ASD could very well also be a wage disturbance. Figure 5 plots its IRFs for $\widetilde{\varepsilon}_{C,t}$ and for $\varepsilon_{w,t}$ in the SW and in our specification with three ASDs and the SW risk premium and investment disturbance excluded. The IRFs of $\varepsilon_{b,t}$ in the two specifications generate a similar business cycle, also quantitatively. A positive shock to $\widetilde{\varepsilon}_{C,t}$ also induces a recession with a reduction in output, investment, and employment. However, it leads to an increase in installed capital and capital services although the latter less than the first. In contrast to the SW $\varepsilon_{w,t}$ shock it goes together with a decrease in the price of capital.

$\widetilde{\varepsilon}_{C,t}$ is an AR(1) process, and the posterior mean of the auto-regressive coefficient is equal to 0.19. The SW $\varepsilon_{w,t}$ disturbance is a very persistent ARMA(1,1) process. The presence of $\widetilde{\varepsilon}_{C,t}$ in the empirical model strongly reduces the coefficient of the MA component of $\varepsilon_{w,t}$, but has little impact on the AR component.[25]

Including $\widetilde{\varepsilon}_{C,t}$ in the empirical specification does not reduce the role of $\varepsilon_{w,t}$ for fluctuations of key variables. $\varepsilon_{w,t}$ remains the most important disturbance for key economic aggregates. The only exception is the wage *growth* rate. In the SW specification $\varepsilon_{w,t}$ explains 61.6% of the volatility of wage growth, whereas it only explains 13.3% in our preferred specification. This role is clearly taken over by $\widetilde{\varepsilon}_{C,t}$ which explains 53.5% of wage growth volatility. $\widetilde{\varepsilon}_{C,t}$ also plays a nontrivial role for fluctuations in the capital stock, capacity utilization, and the rental rate of capital, explaining 9.8%, 14.7%, and 13.1%, of total variability respectively.

The results indicate that this agnostic disturbance increases the wage mark-up and is associated with a lower price of capital and an increased (use of the) capital stock. One

[25]Specifically, with $\widetilde{\varepsilon}_{C,t}$ included in the empirical specification the posterior means of the AR and the MA coefficients of $\varepsilon_{w,t}$ are equal to 0.97 and 0.59, respectively. Estimates with the SW specification for these two numbers are 0.97 and 0.85.

possible explanation is that the increase in the level of used capital (possibly induced by lower prices) comes at the cost of higher wage rates. That is, to operate this larger capital stock, firms have to pay a higher wage rate, perhaps in terms of an overtime premium. The relevant equations are the following:[26]

Agnostic capital-efficiency-wage-mark-up disturbance, $\widetilde{\varepsilon}_{C,t}$

$$\begin{aligned} \text{Valuation:} \quad & q_t = q_1\mathbb{E}_t\left[q_{t+1}\right] + (1-q_1)\mathbb{E}_t\left[r^k_{t+1}\right] \\ & -(\boldsymbol{r_t - \mathbb{E}_t\left[\pi_{t+1}\right]}) - \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{4,C}}\widetilde{\boldsymbol{\varepsilon}}_{\boldsymbol{C,t}}, \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{4,C}} < \mathbf{0}, & (27) \\ \text{Utilization:} \quad & z_t = z_1\left(\Psi\right) r^k_t + \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{7,C}}\widetilde{\boldsymbol{\varepsilon}}_{\boldsymbol{C,t}}, \quad \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{7,C}} > \mathbf{0}, & (28) \\ \text{Capital:} \quad & k_t = k_1\left(\Psi\right) k_{t-1} + \left(1 - k_1\left(\Psi\right)\right) i_t + \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{8,C}}\widetilde{\boldsymbol{\varepsilon}}_{\boldsymbol{C,t}}, \quad \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{8,C}} > \mathbf{0}, & (29) \\ \text{Wage:} \quad & w_t = w_1 w_{t-1} + (1-w_1)(\mathbb{E}_t[w_{t+1} + \pi_{t+1}] \\ & -w_2\pi_t + w_3\pi_{t-1} - w_4\mu_{w,t} + \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{13,C}}\widetilde{\boldsymbol{\varepsilon}}_{\boldsymbol{C,t}}, \quad \widetilde{\boldsymbol{\Upsilon}}_{\boldsymbol{8,C}} > \mathbf{0}. & (30) \end{aligned}$$

4.4. Correlation estimated innovations

Estimated innovations are supposed to be orthogonal to each other and display no auto-correlation. In practice this is often not the case. As shown in Appendix E.3, the ASD system does a substantially better job than the SW system regarding cross-correlations. Both $\widetilde{\eta}_{A,t}$ and $\widetilde{\eta}_{B,t}$ are less correlated with other innovations than their SW counterparts $\eta_{b,t}$ and $\eta_{i,t}$. Moreover, the cross-correlations of the regular structural disturbances that are present in both specifications are also less correlated. Specifically, whereas the SW has nine correlation coefficients that are significantly different from zero at the 10% level for its seven innovations, the ASD system has four significant correlation coefficients for its eight innovations and only two if we exclude the eighth innovation of the ASD system that is associated with $\widetilde{\varepsilon}_{C,t}$.

The ASD specification also does better regarding the auto-correlation of the innovations.

[26]We leave out the overall budget constraint since the role of the disturbance in this equation is very minor, but its impact in this equation is like a contractionary fiscal expenditure shock. w_t is the real wage rate and $\mu_{w,t}$ is the real wage mark-up, i.e., the difference between the wage rate and the marginal rate of substitution between consumption and leisure. Also see footnote 20 for additional information.

In the SW system, four of the estimated innovations display significant auto-correlation at the 10% level. That number reduces to two in the ASD system and one of the significant coefficients is for the innovation associated with the additional disturbance, $\widetilde{\varepsilon}_{C,t}$, for which there is no counterpart in the SW system.

5. Concluding comments

Structural disturbances play a key role in modern business cycles models. Thus, it is important to introduce them correctly. Having wrong formulations will lead to the wrong inference on what type of disturbances matter most for the fluctuations of key economic variables. One of the main objectives of structural models is to do policy analysis. Deriving optimal fiscal and monetary policy correctly also depends crucially on formulating structural disturbances correctly since these are important ingredients of optimal policy rules.

This paper shows that misspecifications can also lead to substantial distortions in parameter estimates and implied model properties. Obviously, the analysis of government policies will be flawed if parameter estimates are incorrect. For example, the impact of monetary policy on economic aggregates in New Keynesian models depend crucially on getting parameters related to the degree of price and wage stickiness right.

The development of MCMC techniques has made it possible to estimate larger models with a larger set of observables. To avoid singularity issues this also requires including more disturbances which enhances the challenge to model them all correctly. ASDs can help. First, they can be used to test whether the specification of a regular structural disturbance is correct and if found problematic can provide insights on how to improve its specification. Researchers can also simply add ASDs to the set of structural disturbances without having any concern about these introducing misspecification.

Focusing on the misspecification of disturbances is only a first step in a proper evaluation of a structural model. Moreover, economists are often more interested in how the model itself magnifies and propagates shocks than in what created the initial disruption. Our procedure is helpful in this regard. By being more agnostic about the nature of structural disturbances one is less likely to distort the analysis of what one is ultimately interested in.

6. Tables and figures

Table 1: Parameter explanations

α	Capital share
σ_c	Inverse IES of consumption
Φ	Fixed cost in production
ϕ	Elasticity of adjustment cost function
λ	Degree of consumption habits
ξ_w	Degree of wage rigidity
σ_ℓ	Inverse IES of leisure
ξ_p	Degree of price rigidity
ι_w	Degree of indexation for wages
ι_p	Degree of indexation for prices
ψ	Elasticity of capital utilization adj. cost function
r_π	Taylor rule coefficient on inflation
ρ	Degree of interest rate smoothing in Taylor rule
r_y	Taylor rule coefficient on output gap
$r_{\Delta y}$	Taylor rule coefficient on change in output gap
	for $j \in \{a, b, g, i, r, p, w\}$:
ρ_j	Persistence of exogenous disturbance j
σ_j	Standard deviation of exogenous disturbance j
	for $j \in \{p, w\}$:
μ_j	MA coefficient of exogenous disturbance j

Notes. The table reports the parameters of the SW model that are estimated and their interpretation. The list of exogenous disturbances is given in the text.

Table 2: Average absolute errors across Monte Carlo experiments

	true value	average error first MC misspecified	agnostic	SW	average error second MC misspecified	agnostic	SW
α	0.19	0.098	0.035	0.028	0.056	0.048	0.037
σ_c	1.39	0.384	0.246	0.191	0.540	0.288	0.226
Φ	1.61	0.217	0.212	0.191	0.192	0.212	0.164
ϕ	5.48	1.793	1.326	0.899	1.429	1.269	0.896
h	0.71	0.096	0.069	0.052	0.083	0.077	0.057
ξ_w	0.73	0.082	0.090	0.076	0.092	0.095	0.081
σ_ℓ	1.92	1.652	0.640	0.532	1.506	0.939	0.831
ξ_p	0.65	0.130	0.074	0.068	0.090	0.080	0.070
ι_w	0.59	0.205	0.165	0.159	0.190	0.168	0.160
ι_p	0.22	0.142	0.109	0.101	0.128	0.112	0.100
ψ	0.54	0.182	0.128	0.109	0.150	0.134	0.118
r_π	2.03	0.295	0.277	0.241	0.347	0.380	0.333
ρ	0.81	0.031	0.025	0.022	0.034	0.038	0.030
r_y	0.08	0.051	0.025	0.021	0.055	0.034	0.029
$r_{\Delta y}$	0.22	0.058	0.014	0.012	0.057	0.039	0.033
ρ_a	0.95	0.071	0.028	0.020	-	-	-
ρ_b	0.18	0.161	0.078	0.073	0.133	0.079	0.071
ρ_g	0.97	0.020	0.016	0.013	0.018	0.016	0.014
ρ_i	0.71	-	-	-	-	-	
ρ_r	0.12	-	-	-	0.089	0.072	0.067
ρ_p	0.90	0.181	0.090	0.067	0.188	0.070	0.053
ρ_w	0.97	0.031	0.030	0.019	0.022	0.029	0.021
μ_p	0.74	0.246	0.188	0.161	0.250	0.173	0.139
μ_w	0.88	0.071	0.072	0.056	0.069	0.071	0.057
σ_a	0.45	0.441	0.061	0.052	-	-	-
σ_b	0.24	0.050	0.021	0.021	0.040	0.023	0.021
σ_g	0.52	0.035	0.027	0.026	0.026	0.027	0.025
σ_i	0.45	-	-	-	-	-	
σ_r	0.24	-	-	-	0.013	0.015	0.014
σ_p	0.14	0.022	0.017	0.015	0.019	0.017	0.015
σ_w	0.24	0.026	0.021	0.020	0.022	0.023	0.021

Notes. This table reports the average absolute error across Monte Carlo replications for the indicated parameter and empirical specification. See Table 1 for the definitions of the parameters. The first (second) Monte Carlo experiment corresponds to the case when the true *dgp* does not include the monetary policy (TFP) disturbance, but the empirical model leaves out the investment disturbance instead.

Table 3: Misspecification tests for the original Smets-Wouters empirical model

structural SW disturbance excluded	ASD added	marginal data density
None (original SW)	no	-922.40
TFP, $\varepsilon_{a,t}$	yes	-931.21
Risk premium, $\varepsilon_{b,t}$	yes	**-908.79**
Government expenditure, $\varepsilon_{g,t}$	yes	-934.14
Investment-specific, $\varepsilon_{i,t}$	yes	**-919.81**
Monetary policy,$\varepsilon_{r,t}$	yes	-926.88
Price mark-up, $\varepsilon_{p,t}$	yes	-938.85
Wage mark-up, $\varepsilon_{w,t}$	yes	-947.31

Notes. The table reports the marginal data density for different empirical specifications. The first row reports the value for the original SW specification. The specifications considered in subsequent rows replace the indicated structural disturbance with an agnostic structural disturbance. The bold numbers indicate the cases for which the MDD is higher when the indicated structural disturbance is replaced by an ASD.

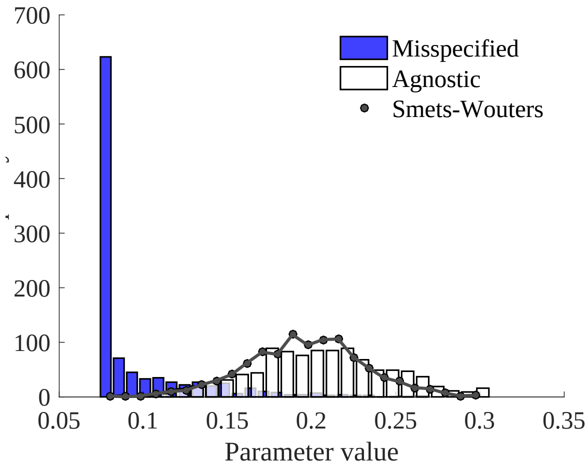
Table 4: Role of structural disturbances for variance

	risk/preference		investment		wage
	SW $\varepsilon_{b,t}$	agnostic $\widetilde{\varepsilon}_{A,t}$	SW $\varepsilon_{i,t}$	agnostic $\widetilde{\varepsilon}_{B,t}$	agnostic $\widetilde{\varepsilon}_{C,t}$
output	1.53	1.14	7.34	2.17	0.28
flex. price output	0	2.08	5.39	1.02	0.36
consumption	2.18	1.51	2.83	0.49	0.25
investment	0.22	1.06	44.2	29.3	1.00
hours	2.52	1.29	8.15	4.97	2.03
capital	0.04	0.12	32.5	2.37	9.75
utilization	0.86	4.14	35.4	9.46	14.7
price of capital	45.4	18.6	36.0	31.6	7.21
marginal cost	0.87	15.2	3.11	2.61	5.13
policy rate	7.40	17.2	18.3	12.5	0.65
inflation	0.58	0.68	3.18	3.96	0.91
output growth	22.1	21.3	15.8	8.04	1.82
consumption growth	61.2	61.7	0.95	2.03	0.10
investment growth	2.46	12.6	82.1	70.0	0.81

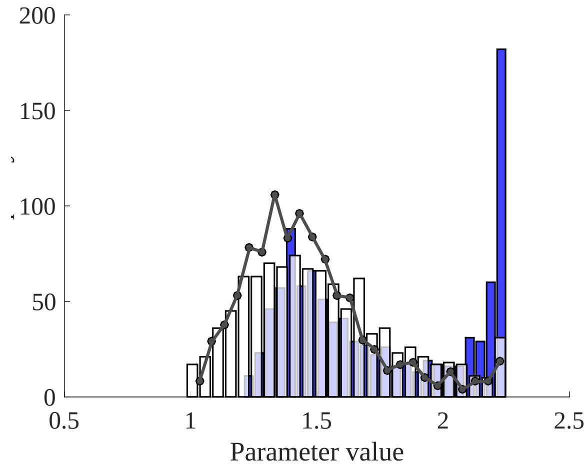
Notes. The table reports the percentage of total variability explained by the SW versus the agnostic risk-premium disturbance, the SW versus the agnostic investment disturbance, and the agnostic wage disturbance. The numbers for the SW disturbance are from estimation of the original SW model. The numbers for the agnostic disturbance are from our preferred empirical model with three ASDs.

Figure 1: Histograms for parameter estimates: First Monte Carlo experiment

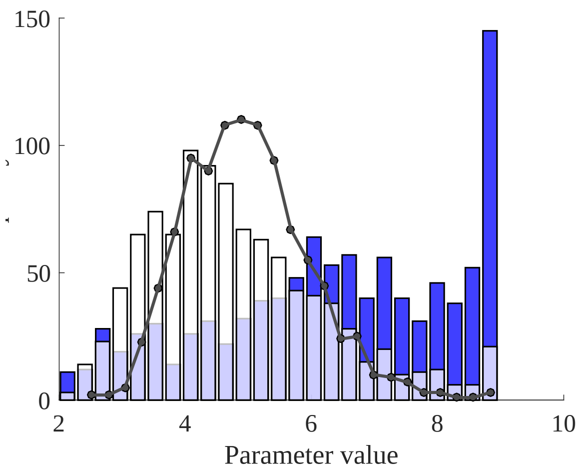
(a) Capital share, α



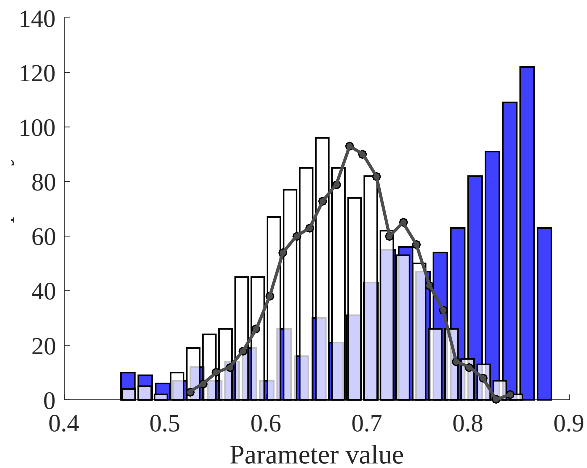
(b) Elasticity intertemporal substitution, σ_c



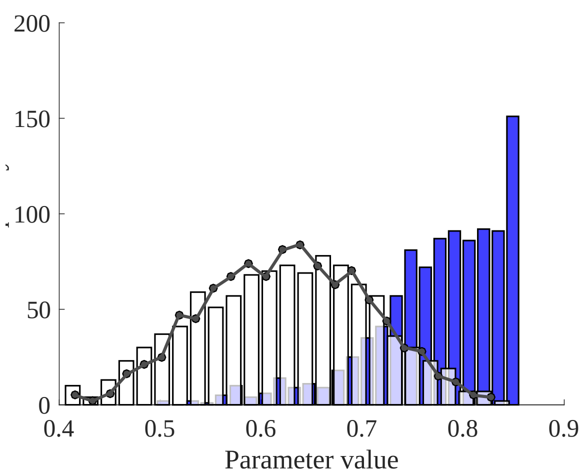
(c) Adjustment cost, ϕ



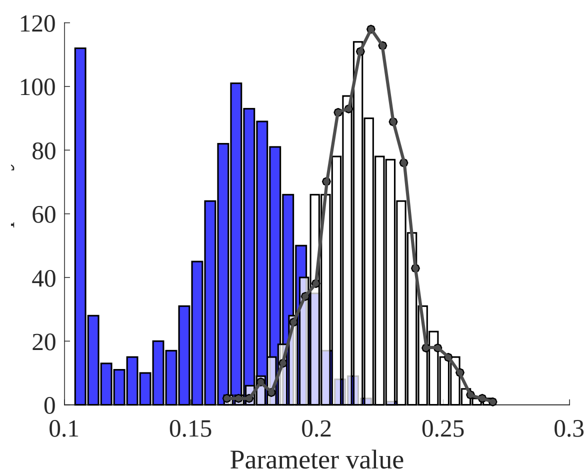
(d) Habit, λ



(e) Price rigidity, ξ_p



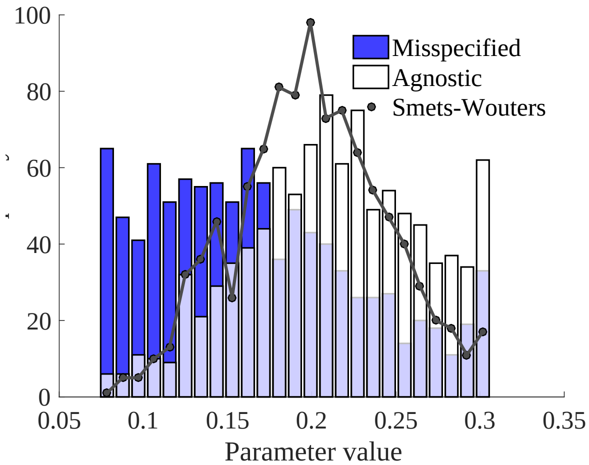
(f) Taylor rule coefficient, $r_{\Delta y}$



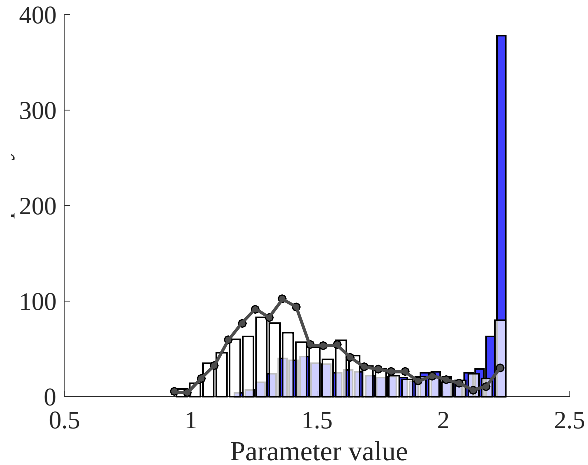
Notes. The panels plot the distribution of the indicated parameter across the Monte Carlo replications. The color of the histograms for the misspecified case changes in a lighter shade when they overlap with the histogram for the agnostic specification. In this experiment, the true *dgp* does not include the monetary policy disturbance, but the empirical model leaves out the investment disturbance instead.

Figure 2: Histograms for parameter estimates: Second Monte Carlo experiment

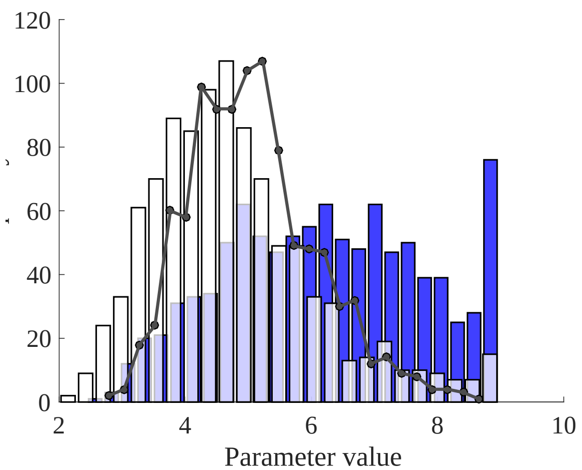
(a) Capital share, α



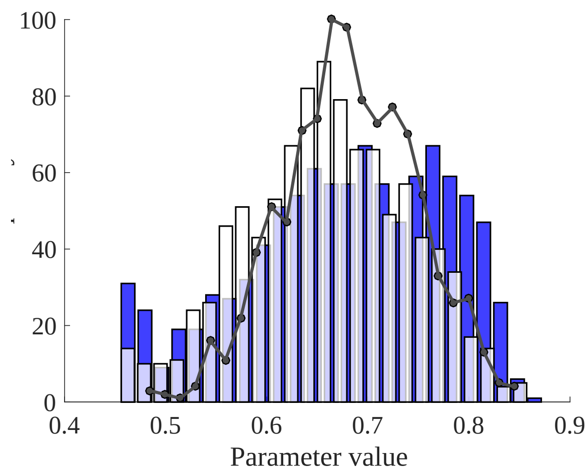
(b) Elastictity intertemporal substitution, σ_c



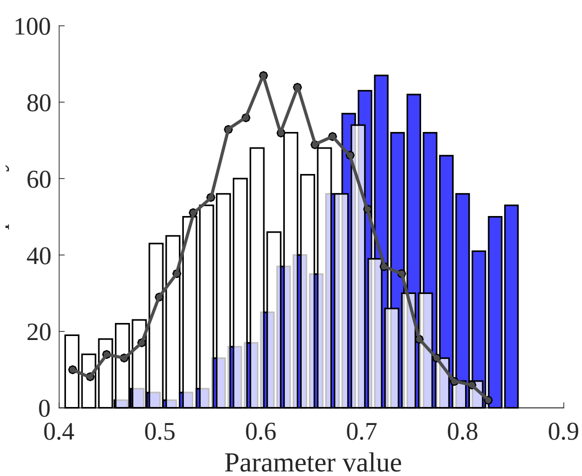
(c) Adjustment cost, ϕ



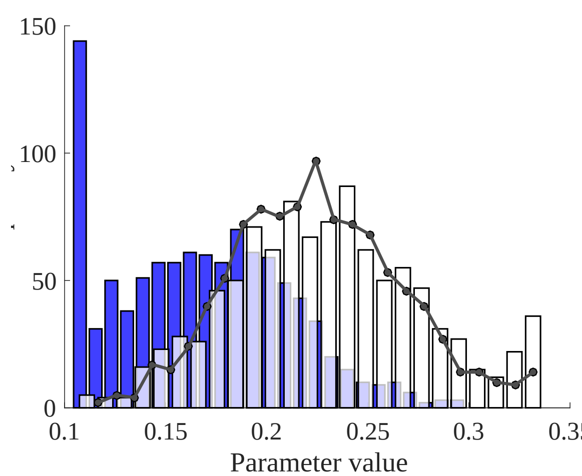
(d) Habit, λ



(e) Price rigidity, ξ_p

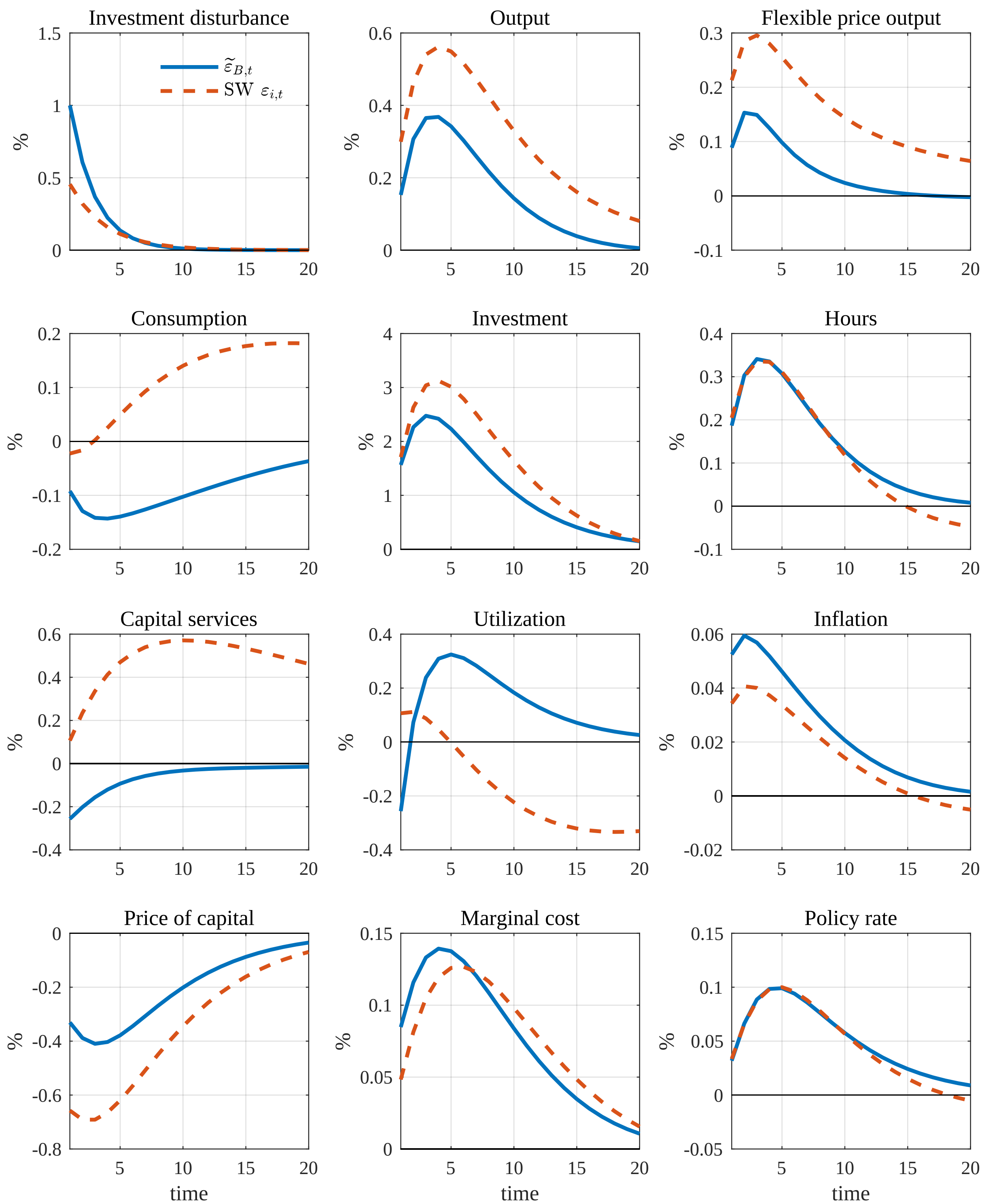


(f) Taylor rule coefficient, $r_{\Delta y}$



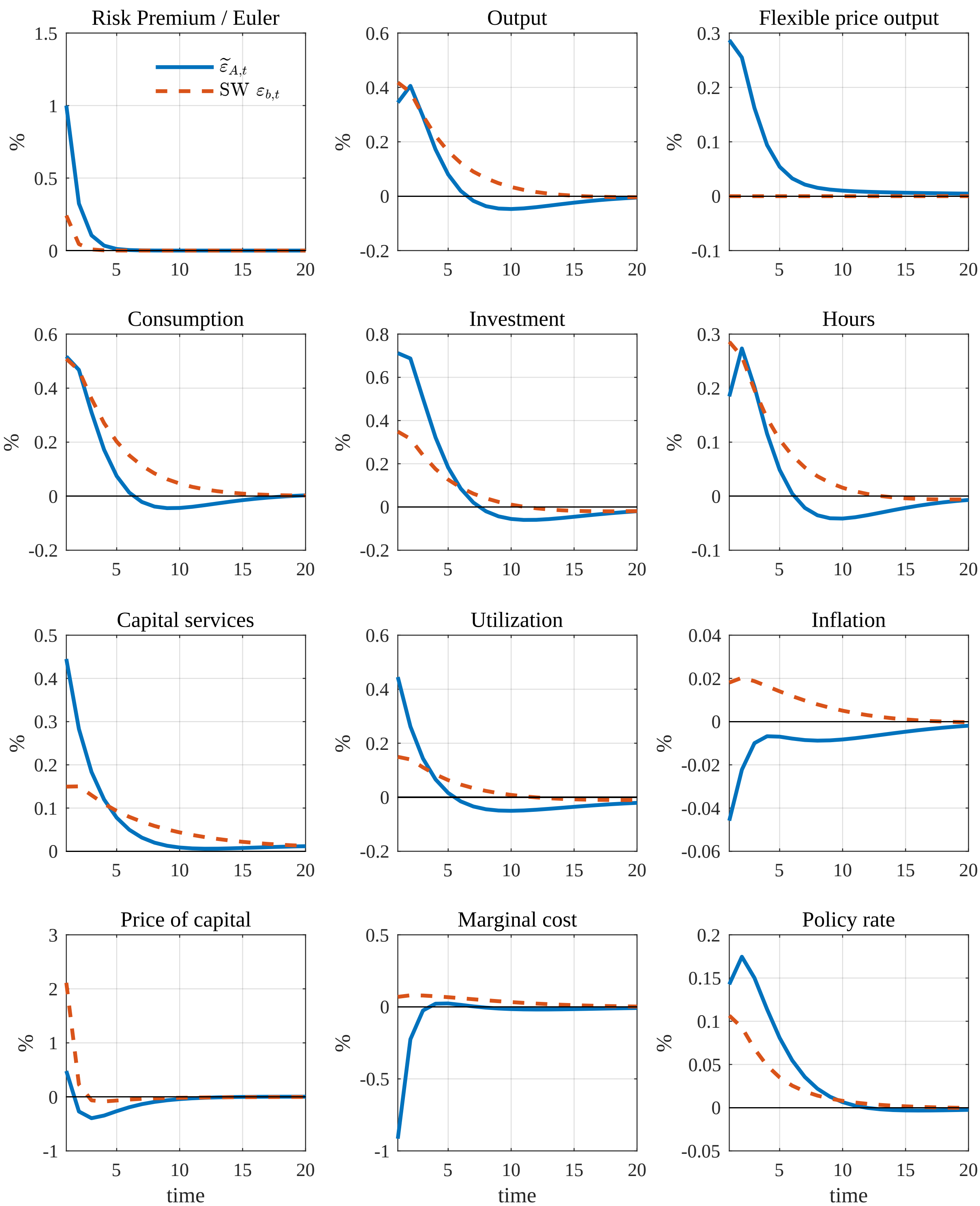
Notes. The panels plot the distribution of the indicated parameter across the Monte Carlo replications. The color of the histograms for the misspecified case changes in a lighter shade when they overlap with the histogram for the agnostic specification. In this experiment, the true *dgp* does not include the TFP disturbance, but the empirical model leaves out the investment disturbance instead.

Figure 3: IRFs of the SW investment and the agnostic investment-modernization disturbance



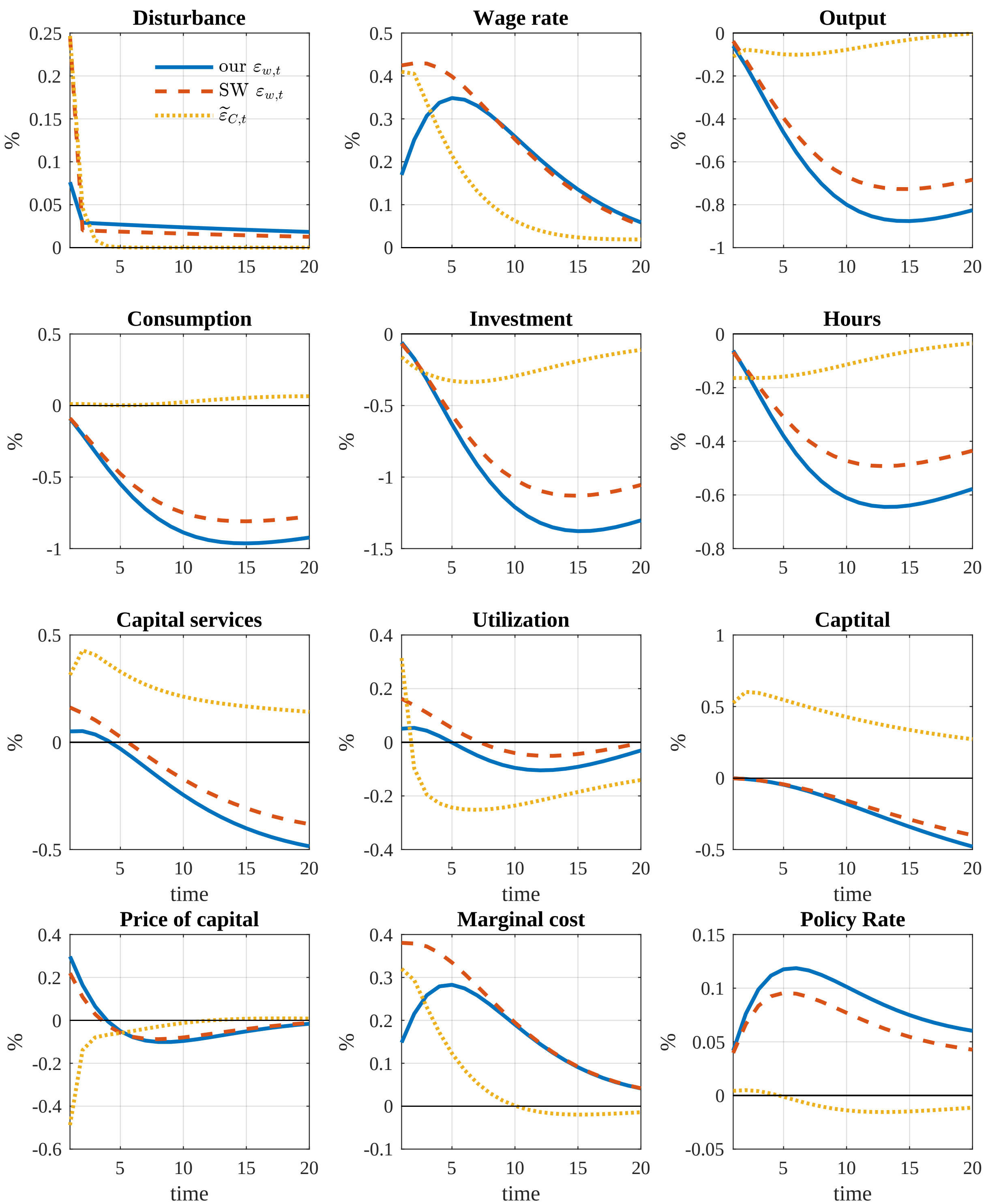
Notes. These panels plot the IRFs of the SW investment-specific productivity disturbance and the agnostic disturbance $\widetilde{\varepsilon}_{B,t}$ that we interpret as an investment-modernization disturbance.

Figure 4: IRFs of the SW risk-premium and the agnostic Euler disturbance



Notes. These panels plot the IRFs of the SW risk-premium disturbance and the agnostic disturbance $\widetilde{\varepsilon}_{A,t}$ that we interpret as an Euler disturbance.

Figure 5: IRFs of the agnostic capital-efficiency wage mark-up disturbance



Notes. These panels plot the IRFs of the agnostic disturbance $\widetilde{\varepsilon}_{C,t}$ that we interpret as a capital-efficiency wage mark-up disturbance. They also plot the SW wage disturbance for the original SW specification and ours.

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Appendix A. The ASD procedure: General setup

Using a simple model, Section 2 made clear that adding an agnostic structural disturbance (ASD) is as simple as adding an exogenous random variable multiplied by a coefficient to *each* model equation. The coefficients are unrestricted and not related to any of the structural parameters of the model. It also shows how the ASD coefficients appear in the model's policy functions and that an alternative way to incorporate ASDs consists of adding them directly to the policy functions. In this appendix, we generalize the discussion.

Appendix A.1. Adding ASDs to model equations

Consider the following linearized model:

$$0_{N\times 1} = \mathbb{E}_t \left[\Lambda_2 (\Psi) s_{t+1} + \Lambda_1 (\Psi) s_t + \Lambda_0 (\Psi) s_{t-1} + \Gamma (\Psi) \varepsilon_{t+1} + \Upsilon (\Psi) \varepsilon_t\right], \tag{A.1a}$$

$$\varepsilon_t = \mathrm{P}\varepsilon_{t-1} + \Omega\eta_t, \tag{A.1b}$$

$$\mathbb{E}_t \left[\eta_{t+1}\right] = 0, \tag{A.1c}$$

$$\mathbb{E}_t \left[\eta_{t+1}\eta'_{t+1}\right] = I_{M\times M}, \tag{A.1d}$$

where Ψ is the vector containing the structural parameters, s_t is the $N \times 1$ vector of endogenous variables, and ε_t is the $M \times 1$ vector of exogenous random variables. All variables are defined relative to their steady state values. Most linearized DSGE models can be represented with such a system of equations. The literature typically assumes that innovations are assumed to be orthogonal to each other, that is, Ω is assumed to be a diagonal matrix. We make the same assumption.[27]

We first discuss the case for which s_t includes only state variables and all N state variables are observables. Suppose that the researcher is only sure about M_1 structural disturbances. These are part of the vector, $\varepsilon_{1,t}$. If $M_1 < N$ and there are no other disturbances, then there is a singularity problem. One option would be to add measurement error. But structural disturbances and measurement errors are very different. Structural disturbances affect economic variables and propagate through the system according to the economic mechanisms of the model. As discussed in the main text, measurement error disturbances do not. Moreover, most researchers would find it undesirable if measurement error "explains" a large part of the data. Another option is to make a best guess and to add a vector $\varepsilon_{2,t}$ with M_2 additional

[27] The literature typically also assumes that P is diagonal. An exception is Cúrdia and Reis (2012).

structural disturbances with $M_2 \geq N - M_1$. Equation (A.1) can then be written as

$$0_{N\times 1} = \mathbb{E}_t \left[\Lambda_2(\Psi) s_{t+1} + \Lambda_1(\Psi) s_t + \Lambda_0(\Psi) s_{t-1} + \Gamma(\Psi)\varepsilon_{t+1} + \Upsilon(\Psi)\varepsilon_t\right]$$

$$= \mathbb{E}_t \left[\begin{array}{c} \Lambda_2(\Psi) s_{t+1} + \Lambda_1(\Psi) s_t + \Lambda_0(\Psi) s_{t-1} \\ + \left[\Gamma_1(\Psi)\ \Gamma_2(\Psi)\right] \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix} + \left[\Upsilon_1(\Psi)\ \Upsilon_2(\Psi)\right] \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \end{array} \right], \tag{A.2a}$$

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} = \begin{bmatrix} \mathrm{P}_{11} & \mathrm{P}_{12} \\ \mathrm{P}_{21} & \mathrm{P}_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} + \begin{bmatrix} \Omega_{11} & 0_{M_1\times M_2} \\ 0_{M_2\times M_1} & \Omega_{22} \end{bmatrix} \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}, \tag{A.2b}$$

$$\mathbb{E}_t \begin{bmatrix} \eta_{1,t+1} \\ \eta_{2,t+1} \end{bmatrix} = 0, \tag{A.2c}$$

$$\mathbb{E}_t \left[\begin{bmatrix} \eta_{1,t+1} \\ \eta_{2,t+1} \end{bmatrix} \begin{bmatrix} \eta_{1,t+1} & \eta_{2,t+1} \end{bmatrix} \right] = I_{M\times M}. \tag{A.2d}$$

The column vectors $\Gamma_2(\Psi)$ and $\Upsilon_2(\Psi)$ capture the restrictions imposed by the M_2 additional structural disturbances.

Introducing ASDs. With ASDs the system of equations is modified as follows:

$$0_{N\times 1} = \mathbb{E}_t \left[\begin{array}{c} \Lambda_2(\Psi) s_{t+1} + \Lambda_1(\Psi) s_t + \Lambda_0(\Psi) s_{t-1} \\ +\Gamma_1(\Psi)\varepsilon_{1,t+1} + \Upsilon_1(\Psi)\varepsilon_{1,t} + \widetilde{\Upsilon}_2\widetilde{\varepsilon}_{2,t} \end{array} \right]. \tag{A.3}$$

As in the main text, all ASD variables and their associated coefficients are denoted with a tilde. As long as $\mathrm{P}_{21} = 0$, then one can exclude $\widetilde{\varepsilon}_{2,t+1}$ from the system, since what matters is $\mathbb{E}_t[\widetilde{\varepsilon}_{2,t+1}] = \mathrm{P}_{22}\widetilde{\varepsilon}_{2,t}$ and the reduced-form coefficient $\widetilde{\Upsilon}_2$ captures this forward looking aspect of ASDs as well as the contemporaneous impact of the ASD on the model equations.[28] Thus, adding an agnostic disturbance introduces one additional parameter for each model equation.[29,30] Replacing regular structural disturbances with agnostic structural disturbances may make it harder to identify Ψ, the structural parameters of the model. As discussed in Appendix C.2, this turned out to be not an issue for the experiments discussed in this paper.

Appendix A.2. Adding ASDs to model solutions

We start this section with a proposition that will be helpful with the second formulation of the ASD procedure. Consider again the model given in Equation (A.2), which divides the vector with exogenous disturbances, ε_t, into two parts, the $M_1 \times 1$ vector, $\varepsilon_{1,t}$, and the $M_2 \times 1$

[28]If $\mathrm{P}_{2,1} \neq 0$, then additional reduced-form coefficients would be needed to capture the predictive power of $\varepsilon_{1,t}$ for future values of $\widetilde{\varepsilon}_{2,t}$. The assumption that $\mathrm{P}_{2,1} = 0$ is weak given that the standard assumption in the literature is that structural disturbances are independent random variables.

[29]Without loss of generality one can set the standard deviations of the innovation of the ASDs equal to 1, which in this case is a normalization of the diagonal elements of $\Omega_{2,2}$. As with regular structural disturbances, one would need to estimate the parameters of the time series specification contained in G.

[30]As discussed in Appendix E, one could choose to leave the agnostic disturbance out of some equations.

vector, $\varepsilon_{2,t}$. A recursive solution to Equation (A.2) has the following form:

$$\begin{aligned} s_t &= A(\Psi)\, s_{t-1} + B(\Psi)\varepsilon_t \\ &= A(\Psi)\, s_{t-1} + \begin{bmatrix} B_1(\Psi) & B_2(\Psi) \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}. \end{aligned} \tag{A.4}$$

The following proposition states that the properties of $\varepsilon_{2,t}$ do *not* affect the coefficients of the policy rule related to s_{t-1} and $\varepsilon_{1,t}$, that is, they do not affect $A(\Psi)$ and $B_1(\Psi)$. Thus, it does not matter whether $\varepsilon_{2,t}$ is a regular or an agnostic structural disturbances and the time series properties of $\varepsilon_{2,t}$ do not matter either. The only assumption needed is that the elements of P_{21} are equal to zero, which corresponds to the case when $\varepsilon_{1,t}$ has no effect on *future* values of $\varepsilon_{2,t}$. This is not very restrictive given that the literature usually sets all elements of P_{21} equal to zero (and also all elements of P_{12}, $\Omega_{1,2}$, and $\Omega_{2,2}$ as well as the off-diagonal elements of P_{11}, P_{22}, $\Omega_{1,1}$ and $\Omega_{2,2}$).

Proposition 1. *If the model is given by equation (A.2) and all elements of* P_{21} *are equal to zero, then (i)* $A(\Psi)$ *and* $B_1(\Psi)$ *do not depend on* $\Gamma_2(\Psi)$ *and* $\Upsilon_2(\Psi)$*, which characterize the nature of the additional disturbances, and (ii)* $A(\Psi)$ *and* $B_1(\Psi)$ *do not depend on* P_{22}*,* Ω_{21}*, and* Ω_{22}*, which characterize the time series properties of* $\varepsilon_{2,t}$*.*

Proof. Substitution of the policy rule as given in Equation (A.4) into the system of Equations (A.2) gives,

$$0_{N\times 1} = \left(\Lambda_2 A^2 + \Lambda_1 A + \Lambda_0\right) s_{t-1} + \left(\Lambda_2 AB + \Lambda_2 B\mathrm{P} + \Lambda_1 B + \Gamma\mathrm{P} + \Upsilon\right)\varepsilon_t, \tag{A.5}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} \end{bmatrix}', \tag{A.6}$$

$$B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}, \tag{A.7}$$

$$\mathrm{P} = \begin{bmatrix} \mathrm{P}_{11} & \mathrm{P}_{12} \\ \mathrm{P}_{21} & \mathrm{P}_{22} \end{bmatrix}, \tag{A.8}$$

where we have suppressed the dependence of coefficients on Ψ. The first equation has to hold for all values of s_{t-1} and ε_t. This implies that a solution must satisfy

$$\Lambda_2 A^2 + \Lambda_1 A + \Lambda_0 = 0_{N\times N} \tag{A.9}$$

and

$$\Lambda_2 AB + \Lambda_2 BG + \Lambda_1 B + \Gamma\mathrm{P} + \Upsilon = 0_{N\times(M_1+M_2)}. \tag{A.10}$$

A does not depend on the time series properties of $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, since B, P, and Ω do not appear in Equation (A.9). Equation (A.10) can be written as follows

$$\overline{\Lambda}\begin{bmatrix} B_1 & B_2 \end{bmatrix} + \Lambda_2 \begin{bmatrix} B_1 & B_2 \end{bmatrix}\begin{bmatrix} \mathrm{P}_{11} & \mathrm{P}_{12} \\ \mathrm{P}_{21} & \mathrm{P}_{22} \end{bmatrix} + \Gamma\begin{bmatrix} \mathrm{P}_{11} & \mathrm{P}_{12} \\ \mathrm{P}_{21} & \mathrm{P}_{22} \end{bmatrix} + \Upsilon = 0_{N\times(M_1+M_2)}, \tag{A.11}$$

where $\overline{\Lambda} = \Lambda_2 A + \Lambda_1$. This is a system of $N \times (M_1 + M_2)$ equations to solve for the elements of B. It can be split into the following two sets of systems:

$$\overline{\Lambda} B_1 + \Lambda_2 B_1 \mathrm{P}_{11} + \Lambda_2 B_2 \mathrm{P}_{21} + \Gamma_1 \begin{bmatrix} \mathrm{P}_{11} \\ \mathrm{P}_{21} \end{bmatrix} + \Upsilon_1 = 0_{N \times M_1}, \tag{A.12}$$

$$\overline{\Lambda} B_2 + \Lambda_2 B_1 \mathrm{P}_{12} + \Lambda_2 B_2 \mathrm{P}_{22} + \Gamma_2 \begin{bmatrix} \mathrm{P}_{12} \\ \mathrm{P}_{22} \end{bmatrix} + \Upsilon_2 = 0_{N \times M_2}. \tag{A.13}$$

If $\mathrm{P}_{21} = 0$, then Equation (A.12) contains $N \times M_1$ equations to solve for all the elements of B_1. The solution cannot depend on P_{22} or Ω_{22} since these matrices do not appear in this equation.■

It is intuitive that the elements of P_{21} have to be equal to zero, that is, $\varepsilon_{1,t}$ should not affect future values of $\varepsilon_{2,t}$. If current values of $\varepsilon_{1,t}$ do affect future values of $\varepsilon_{2,t}$ and therefore future values of s_t, then one has to know how $\varepsilon_{2,t}$ affects model outcomes to determine how $\varepsilon_{1,t}$ affects current outcomes for s_t.

Introducing ASDs. The solution to a linearized model can be written as:

$$s_t = \sum_{m=1}^{M} s_t^{[m]}, \tag{A.14}$$

$$s_t^{[m]} = A(\Psi)\, s_{t-1}^{[m]} + B_{\cdot,m}(\Psi)\, \varepsilon_{m,t}, \tag{A.15}$$

where $s_t^{[m]}$ represents the outcome of the state variable if the *only* disturbance in the economy is the m^{th}-disturbance, $\varepsilon_{m,t}$, and $B_{\cdot,m}$ is the m^{th} column of B. Thus, one can think of the s_t variables as the sum of the outcomes in "one-disturbance" economies. The linearity of the model is important for this additive property. According to Proposition 1, the coefficients on the lagged state variable, $A(\Psi)$, do not depend on the particular disturbance considered. That is, whereas $B_{\cdot,m}(\Psi)$ is indexed by m because it depends on what kind of disturbance is the driving force of the economy, $A(\Psi)$ does not. This property greatly increases the efficiency of our procedure.

Our proposed procedure consists of including $\widetilde{M}_2$ agnostic structural disturbances. This results in the following time series representation of the policy functions:[31]

$$s_t = \sum_{m=1}^{M} s_t^{[m]}, \tag{A.16a}$$

$$s_t^{[m]} = A(\Psi)\, s_{t-1}^{[m]} + B_{\cdot,m}(\Psi)\, \varepsilon_{m,t} \quad \text{for } m \leq M_1, \tag{A.16b}$$

$$s_t^{[m]} = A(\Psi)\, s_{t-1}^{[m]} + \widetilde{B}_{\cdot,m} \widetilde{\varepsilon}_{m,t} \qquad \text{for } M_1 + 1 \leq m \leq M_1 + \widetilde{M}_2 = M. \tag{A.16c}$$

[31]Proposition 1 indicates that this specification is valid as long as the elements of P_{12} are equal to zero, which is usually the case.

In terms of notation, $B_{\cdot,m}(\Psi)$ contains coefficients associated with a regular structural disturbance which are a function of Ψ and $\widetilde{B}_{\cdot,m}$ contains reduced-form coefficients associated with a structural agnostic disturbance. The *only* difference between this specification and the standard DSGE specification with only regular structural disturbances is that the $\widetilde{B}_{\cdot,m}$ coefficients are unrestricted reduced-form coefficients. Since our agnostic disturbances are structural disturbances, their impact propagates through the system exactly as regular structural disturbances do, that is, as described by $A(\Psi)$. The property of linear models that $A(\Psi)$ does not depend at all on what is the nature of the structural disturbances nor on their time series properties makes it possible to efficiently add structural disturbances to the specification without having to be specific on what they are.

The dimension of $\widetilde{B}_{\cdot,m}$ is equal to N, the number of state variables. This means that adding an agnostic disturbance means estimating an additional N parameters. The number of additional parameters to be estimated is limited because structural disturbances differ in their initial impact, but their propagation through time is the same for all disturbances and controlled by $A(\Psi)$. Moreover, an increase in the standard deviation of an agnostic structural disturbance affects the model variables in exactly the same way as an identical proportional increase of the elements of $\widetilde{B}_{\cdot,m}$. Consequently, the standard deviation of an agnostic disturbance can be normalized to equal 1.[32]

Adding observation equations. If there are observables that are not state variables, then one also needs additional equations for these y_t variables, which for our set-up is given by

$$\begin{aligned} y_t &= \sum_{m=1}^{M} y_t^{[m]}, && \text{(A.17a)} \\ y_t^{[m]} &= C(\Psi)\, s_{t-1}^{[m]} + D_{\cdot,m}(\Psi)\, \varepsilon_{m,t} \quad \text{for } m \leq M_1, && \text{(A.17b)} \\ y_t^{[m]} &= C(\Psi)\, s_{t-1}^{[m]} + \widetilde{D}_{\cdot,m} \widetilde{\varepsilon}_{m,t} \quad \text{for } M_1 + 1 \leq m \leq M_1 + \widetilde{M}_2 = M, && \text{(A.17c)} \end{aligned}$$

where y_t is the $(\overline{N} \times 1)$ vector with observables that are not state variables. Each additional observable used in the estimation will introduce one more coefficient related to the agnostic structural disturbances.

Unobserved components. The system represented in Equation (A.16) makes clear that ASDs can be interpreted simply as unobserved components that are added to a theoretical block. Using reduced-form systems with unobserved components is common practice in macroeconomic time-series models. The data will provide information about the $\widetilde{B}_{\cdot,m}$ (and $\widetilde{D}_{\cdot,m}$) coefficients. If one mainly cares about getting good time-series representations of

[32] If the time series processes of the two disturbances have the same number of parameters, then replacing a regular structural disturbance by an agnostic disturbance typically means estimating an additional $N-1$ parameters.

macroeconomic variables, then can just use this way of incorporating ASDs. However, as shown in our empirical applications, ASDs become much more interesting if one knows the $\widetilde{\Upsilon}_{2,m}$ coefficients. Then one can much better understand how these unobserved components affect the model economy.

Appendix B. Misspecification: Literature review

Most empirical papers that estimate a dynamic macroeconomic model do not raise the issue of model uncertainty or misspecification, except possibly with some robustness exercises.[33] This does – of course – not mean that the profession is not aware that misspecification is a serious concern. In fact, some of the most prominent researchers in this research area have drawn attention to the risk of misspecification. The first subsection discusses evidence that indicates that misspecification of DSGE models is a serious concern. The second subsection discusses approaches proposed in the literature to deal with misspecfication. See Paccagnini (2017) for a more detailed survey.

Appendix B.1. Indications of DSGE misspecification

Del Negro et al. (2007) develop a procedure that allows the data to determine the usefulness of a DSGE model relative to a much less restricted VAR. Using a model very similar to the DSGE model of Smets and Wouters (2003), they find that their procedure does put some weight on the DSGE model, which implies that the restrictions of the DSGE model are of some value. However, they also argue that misspecification is a concern that *"... is not small enough to be ignored."* Using the same methodology, Del Negro and Schorfheide (2009) also find *"... strong evidence of DSGE model misspecification."*

There is also more indirect evidence that misspecification of estimated DSGE models is substantive. Using the Smets and Wouters (2003) model for the Euro Area, Beltran and Draper (2015) find that the data prefer implausible estimates for several parameters. For example, *most* of the mass of the marginal likelihood for the parameter of relative risk aversion is above 200, way above the range of values considered reasonable. This information provided by the likelihood is typically not revealed in empirical studies, since only properties of the posterior are reported and the choice of prior ensures that these aspects of the empirical likelihood have little or no weight in the posterior. A similar conclusion can be drawn from Onatski and Williams (2010). They estimate the same model using uniform priors over bounded ranges. These ranges are such that the priors are less informative than the ones typically used in the literature. Consistent with the results in Beltran and Draper (2015), several of the point estimates in Onatski and Williams (2010) are at the prior bounds. Using a new algorithm to deal with the complexity of estimating likelihood functions, Mickelsson (2015) re-estimates the model of Smets and Wouters (2007) and he also finds that several

[33] Interestingly, there are quite a few macroeconomic models in which agents – especially agents setting fiscal and monetary policy – face model uncertainty. If policy makers face model uncertainty about the correct model, then researchers are likely to do so as well.

parameter estimates are significantly different from the ones reported in Smets and Wouters (2007).

Another possible reason for misspecification is the assumption that parameters are constant. To get efficient estimates we would like to use long time-series data, but the longer the time series the less likely that all parameters are constant. Canova et al. (2015) address this issue and document that this is important for the model of Gertler and Karadi (2010).[34]

Appendix B.2. Dealing with misspecification: Other approaches

Richer models. Exogenous random disturbances are typically assumed not to be correlated with each other. This is a convenient assumption, because allowing for interaction between the different exogenous disturbances would substantially increase the number of parameters to be estimated given that DSGE typically have a several exogenous disturbances. However, it seems quite plausible that such disturbances are correlated. Del Negro and Schorfheide (2009) and Cúrdia and Reis (2012) deal with this possible misspecification and allow for more general processes to describe the behavior of the exogenous random disturbances.

Cúrdia and Reis (2012) find that this generalization has nontrivial consequences for the properties of the model. For example, the impact of a monetary policy shock on output is only half as big when the exogenous random variables are allowed to be correlated and the medium-term impact of a government spending shock switches from being positive to negative.[35]

Enriching a model by allowing for additional features and more general specifications is likely to reduce misspecification. However, richer models typically have more parameters, which will reduce the efficiency of the estimation by reducing the number of degrees of freedom.

Multiple models. Another way to deal with potential misspecification is to consider a set of different DSGE models. These could be compared informally or formally using, for example, relative marginal likelihoods or model averaging.[36] However, given the difficulty of modeling macroeconomic phenomena, it seems likely that *all* models in a set of DSGE models are subject to at least some type of misspecification.

Combining structural and reduced-form models. Ireland (2004) is an early paper that proposes a more general procedure to deal with possible misspecification when estimating a DSGE model even though the word misspecification is not used in the paper. Specifically, Ireland (2004) *"... augments the DSGE model so that its residuals – meaning the movements*

[34] The literature cited in Canova et al. (2015) documents that this is an issue in a variety of DSGE models.

[35] Cúrdia and Reis (2012) still impose the standard assumption that the *innovations* of the shocks are uncorrelated.

[36] See chapter 5 in An and Schorfheide (2007) for a detailed discussion.

in the data that the theory cannot explain – are described by a VAR." To understand this procedure, consider the following representation of the linearized solution of a DSGE model:

$$
\begin{aligned}
s_t &= As_{t-1} + B\eta_t, && \text{(B.1)} \\
y_t &= Cs_{t-1} + D\eta_t, && \text{(B.2)}
\end{aligned}
$$

where s_t is a vector containing (endogenous and exogenous) state variables, y_t is a vector containing the observables, and η_t is a vector containing the innovations of the exogenous random variables. Ireland (2004) proposes to augment the observation Equation (B.2) as follows:

$$y_t = Cs_{t-1} + D\eta_t + u_t \tag{B.3a}$$

$$u_t = Fu_{t-1} + \xi_t \tag{B.3b}$$

where u_t captures the misspecification or incompleteness of the DSGE model. In his application, the structural equations are the policy rules from a standard Real Business Cycle (RBC) model with total factor productivity (TFP) as the only driving process. If the standard deviation of η_t is equal to 0, then this procedure boils down to estimating a standard VAR.

Note that the presence of the "missing elements" that are captured by u_t is assumed to have no effect on that part of agents' behavior that is described by the DSGE model, that is, the matrices A, B, C, and D. For this to be correct it must be true that the response of the economy to a TFP shock does not depend on the presence of other disturbances. One might think that such independence of a DSGE's policy rule to the presence of other disturbances is only correct if the additional disturbances represent measurement error.[37] However, this "independence" property is correct in linear(ized) models in the sense that the specification of the structural part given in Equations (B.1) and (B.2) does not depend on the presence of not included structural disturbances. It must be noted that the assumption that u_t follows a first-order (or even a finite-order) VAR could very well be restrictive. Thus the reduced-form specification for u_t could be misspecified as well.

The most comprehensive methodology to deal with misspecified DSGE models is put forward in Del Negro et al. (2007). Their starting point is a VAR specification of the observables. That is,

$$y_t = \sum_{k=1}^{K} F_k y_{t-1} + G\xi_t \tag{B.4a}$$

$$\mathbb{E}\left[\xi_t \xi_t'\right] = I. \tag{B.4b}$$

The key idea of the DSGE-VAR estimation proposed in Del Negro et al. (2007) is to estimate

[37] Although Ireland (2004) does not refer to the residual between model and data as measurement error, other papers in the literature describing his procedure do. Examples are Del Negro and Schorfheide (2009) and Cúrdia and Reis (2012).

this time series process with the prior distribution for F and Ω that is centered at the values implied by a DSGE model, $F(\Psi)$ and $G(\Psi)$, where Ψ is the vector containing the parameters of the DSGE model. The estimation procedure consists of jointly estimating Ψ, the structural parameters of the DSGE model, which pin down the prior for the VAR coefficients, and the VAR coefficients themselves.

The precision of the prior of the VAR coefficients is controlled with a scalar parameter, λ. If λ is equal to ∞, then one estimates an unrestricted VAR and if λ is equal to 0, then the procedure boils down to estimating a DSGE without allowing for misspecification. The estimation is executed for different values of λ. To determine the optimal value for λ, the authors propose using the marginal data density, which compares in-sample fit with model complexity. If the restrictions imposed by the DSGE model are incorrect, then the procedure will put more weight on the VAR.

As pointed out in Chari et al. (2008), DSGE models often do not imply a VAR representation with a finite number of lags, unless all state variables are included. Thus, not only the DSGE, but also the VAR component of the DSGE-VAR procedure could be misspecified.

Wedges. Yet another approach to deal with misspecification is to add "wedges" to specific model equations. This procedure was introduced in Chari et al. (2007). Inoue et al. (2015) use this setup to formally test for model misspecfication. A wedge may have different interpretations or possibly no simple interpretation. From an econometric point a view, wedges are not different from regular structural disturbances in how they affect time series properties of the model. That is, they impose restrictions on the policy functions just as regular structural disturbances do and it matters crucially how one enters wedges. For example, the assumption that a wedge only enters one and not all model equations is a restriction. Although some wedges can enter more than one equation, wedges used in the literature only enter a few specific model equations chosen by the researcher a priori and – as pointed out in Inoue et al. (2015) – wedges can be introduced in many different ways. By contrast, ASDs appear in all equations. If one prefers a more concise specification, then the idea of our agnostic procedure indicates involves using a statistical model selection criterion not economic arguments.[38]

Appendix C. Additional discussion for Monte Carlo experiments

This appendix starts by giving some additional information about our experiments and continues by providing additional results.

[38] There may be valid economic or other reasoning to make additional restrictions. For example, in our empirical application we give such motivation for our assumption that the ASDs appear in the flexible-price block of the SW with the same coefficient as in the corresponding equation of the actual model. However, such restrictions make the procedure less agnostic.

Appendix C.1. Details

We follow SW and do not estimate the depreciation rate, δ, the steady-state wage markup, $\overline{\mu}$, the steady-state level of government expenditures, $\overline{g}$, the curvature in the Kimball goods-market aggregator, ε_p, and the curvature in the Kimball labor-market aggregator, ε_w. Since we use demeaned data, we also fix the trend growth rate, $\overline{\gamma}$, the parameter controlling steady state hours, $\bar{l}$, the parameter controlling steady state inflation, $\overline{\pi}$, and the discount factor, β.

We deviate in one aspect from SW and that is related to the parameter ρ_{ga}, which captures the impact of the TFP structural disturbance on the government expenditures structural disturbance. We set this coefficient equal to zero in both the true *dgp* and in the empirical model. This implies that all structural disturbances are uncorrelated. This is a typical assumption and makes our misspecification experiment more transparent. As discussed below, the misspecification considered is related to the specification of the set of structural disturbances. If $\rho_{ga} \neq 0$, then we would have to make additional choices whenever the misspecification involves either the TFP or the government spending shock. We explored some alternative cases in which $\rho_{ga} \neq 0$ and found similar results.

We adopt Maximum Likelihood estimation. This involves a nontrivial optimization given the complexity of the model and number of parameters to be estimated. However, our optimization problem is relatively well defined. The empirical model is very close to the true *dgp*. Moreover, we use the true parameter values as the initial conditions for the optimization routine and we specify bounds for the parameter values. These choices decrease computing time and also give a misspecified model the best possible chance to deliver estimates that are close to the truth. The innovation standard deviations of the disturbances are restricted to be in the interval $[0, 10]$ and the coefficients of their time series process in the interval $[0, 99]$. Given our focus on misspecified disturbances, we want these intervals to be large. For the structural parameters we set the lower bound and the upper bound to the first and ninety-ninth percentile according to the SW prior, centered at the parameter values of the true *dgp*.

Although the Monte Carlo experiments focus on ML estimation, we used a Bayesian model comparison for the exercise where we answered whether a researcher would *in practice* reject the wrong empirical specification when compared with the true one. We used a Bayesian approach because Bayesian estimation is the dominant strategy to estimate DSGE models. For this comparison we restricted ourselves to 100 Monte Carlo replication, since each replication involves a computationally intensive MCMC procedure to trace the posterior.

Our analysis has some features in common with Ferroni et al. (2015), but there are important differences. They only consider one specific misspecified empirical model whereas we consider a total of forty-two. Although they consider a limited Monte Carlo experiment (with 100 replications), the main discussion focuses on particular sample of 200 observations. Most importantly, their main focus is on the consequences of using an inverse gamma prior for parameters that could well be zero. Our focus is on the misspecification of the empirical model, not the specification of the prior.

Appendix C.2. Identification of structural parameters

To properly evaluate the misspecification experiments of this paper, it is important that the estimated parameters are identified. If not, then any detected distortions of parameter estimates and implied model properties could be due to lack of identification and not misspecification. We use the test proposed in Komunjer and Ng (2011) to check whether the parameters of the empirical specifications used in our experiments are identified. We will refer to this test as the KN test. This test provides both necessary and sufficient conditions for local identification under a set of weak conditions.[39] It focuses on the state-space representation of the model and – in contrast to earlier identification tests – does not require the user to specify a set of particular autocovariances.[40] The results document that parameters are identified in all experiments. In Appendix D.3, we document that weak identification is not an issue either.

Identification of original Smets-Wouters estimation exercise. SW fix the values of five parameters: depreciation, δ, steady-state wage mark-up, $\overline{\mu}$, steady-state exogenous spending, $\overline{g}$, curvature in the Kimball goods-market aggregator, ε_p, and curvature in the Kimball labor-market aggregator, ε_w. Komunjer and Ng (2011) consider the identification of the SW model, but their empirical specification is slightly different from the one of SW in that all variables are demeaned. By contrast, the data in the SW estimation exercise does contain information about the level, since the inflation rate and the nominal interest rate are in levels. Komunjer and Ng (2011) show that several subsets of the five parameter restrictions mentioned above are sufficient to obtain identification *if* the parameter controlling steady state hours, $\bar{l}$, and the parameter controlling steady state inflation, $\overline{\pi}$, are fixed as well. It makes sense that identification requires more restrictions when information about the levels is not used in the estimation.

Identification of our specifications. The empirical and true specifications used in our Monte Carlo experiments have six structural disturbances, whereas the original SW empirical model has seven. This may imply that less parameters are identified. It is important that the parameters that we try to estimate are identified. If parameters are not identified, then different parameter combinations lead to the same criterion of fit used in the estimation, so it would not be surprising if parameter estimates are different for slightly different specifications.

Consequently, we adopt the following conservative strategy to ensure identification. The KN test checks rank conditions of matrices and to see whether there is a singularity one needs to choose a tolerance criterion. We set the criterion at a level that is more strict than the one chosen by KN.[41] We follow SW and fix the values of the five parameters mentioned above. In addition, we fix all parameters that have a direct effect on the means of variables, since we use demeaned variables in the estimation. The associated parameters are the trend

[39] These are a stability condition and regularity conditions on the innovations.

[40] An example of such an earlier test is Iskrev (2010).

[41] We set "Tol" equal to 1e-2 instead of 1e-3 (a higher number means that the test is more difficult to pass).

Table C.5: Komunjer and NG identification test.

required number		41	225	36	302	-
	n	$\overline{\Delta}_{\Lambda}^{S}$	$\overline{\Delta}_{T}^{S}$	$\overline{\Delta}_{U}^{S}$	$\overline{\Delta}^{S}$	pass?
$\varepsilon_{a,t}$ excluded	12	41	225	36	302	yes
$\varepsilon_{b,t}$ excluded	12	41	225	36	302	yes
$\varepsilon_{g,t}$ excluded	12	41	225	36	302	yes
$\varepsilon_{i,t}$ excluded	12	41	225	36	302	yes
$\varepsilon_{r,t}$ excluded	12	41	225	36	302	yes
$\varepsilon_{w,t}$ excluded	13	41	225	36	302	yes
$\varepsilon_{p,t}$ excluded	13	41	225	36	302	yes

Notes. Here, n is the number of restrictions, which includes the number of coefficients fixed in all experiments and the number of coefficients in the law of motion of the excluded exogenous random variable that are all set to zero. $\overline{\Delta}_{\Lambda}^{S}$ is a matrix that contains the derivatives of all the vectorized elements in the state-space representation of the model (the A, B, C, D matrices and the variance-covariance matrices) evaluated at the true parameter values. It is intuitive that this matrix needs to have full rank for identification. But it is not sufficient. $\overline{\Delta}_{T}^{S}$ and $\overline{\Delta}_{U}^{S}$ are matrices with particular elements related to the state-space representation. The matrix $\overline{\Delta}^{S} = [\overline{\Delta}_{\Lambda}^{S}\ \overline{\Delta}_{T}^{S}\ \overline{\Delta}_{U}^{S}]$ needs to have full rank to pass the KN test.

growth rate, $\overline{\gamma}$, the parameter controlling steady state hours, $\bar{l}$, the parameter controlling steady state inflation, $\overline{\pi}$, and the discount factor, β.[42] Finally, as discussed in Section 2.1, we fix the spillover from the productivity disturbance to exogenous spending and set it equal to zero.

The results of the KN test are reported in Table C.5 and it indicates that the identification test is passed in all cases. That is, lack of identification is not driving the results in this paper.

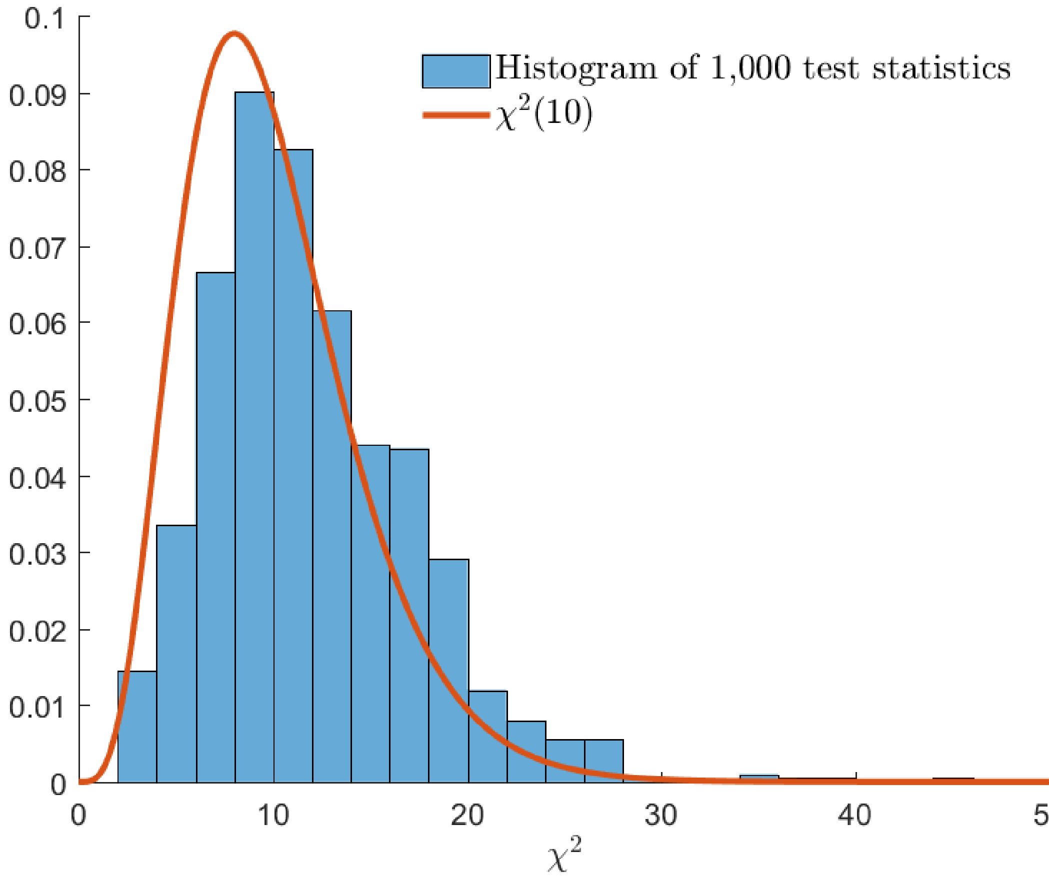
Appendix C.3. Additional results

Figures C.6 and C.7 plot the histograms of the estimated χ^2 statistics across Monte Carlo replications for the two experiments of Section 3 together with the theoretical (large-sample) χ^2 distribution. The number of degrees of freedom is equal to 10. They document that the distribution of test statistics across Monte Carlo replications is quite close to the theoretical one.

Tables C.6 and C.7 document detailed information on the distribution of parameter estimates for the two Monte Carlo experiments.

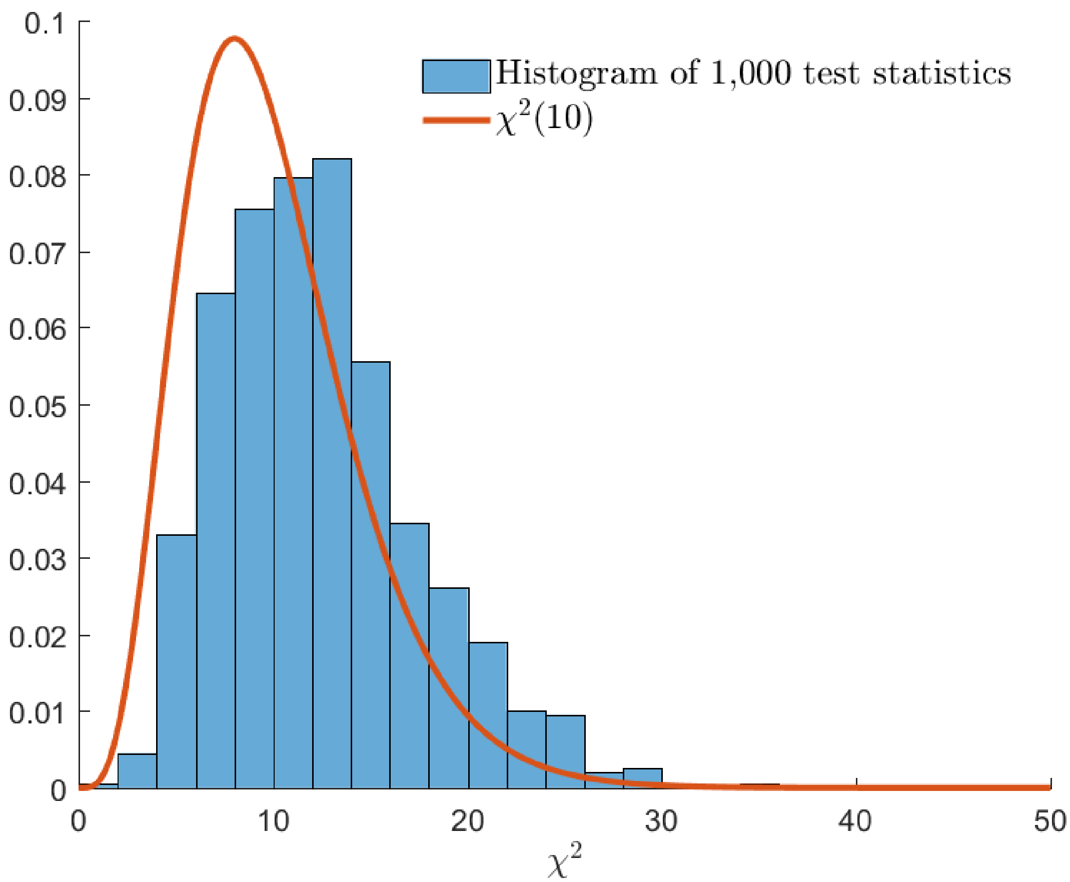
[42] It is a conservative choice to fix all four, since identification only requires that two parameters are fixed according to the test of Komunjer and Ng (2011).

Figure C.6: Likelihood ratio test agnostic versus fully-specified model: First experiment



Notes. The figure plots the distribution of χ^2 statistics of the first Monte Carlo experiment and the theoretical distribution according to large sample theory. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a monetary policy disturbance, but the empirical model leaves out the investment disturbance instead.

Figure C.7: Likelihood ratio test agnostic versus fully-specified model: Second experiment



Notes. The figure plots the distribution of χ^2 statistics of the first Monte Carlo experiment and the theoretical distribution according to large sample theory. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a TFP disturbance, but the empirical model leaves out the investment disturbance instead.

Table C.6: Parameter estimates across Monte Carlo replications: First experiment

				misspecified estimation					ASD procedure					SW specification				
	Truth	**LB**	**UB**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**
α	0.19	0.07	0.31	0.07	0.07	0.08	0.10	0.14	0.15	0.18	0.21	0.23	0.26	0.15	0.17	0.20	0.22	0.24
σ_c	1.39	0.53	2.25	1.35	1.44	1.66	2.16	2.24	1.16	1.29	1.47	1.69	1.95	1.19	1.29	1.42	1.58	1.81
Φ	1.61	1.33	1.89	1.46	1.69	1.86	1.89	1.89	1.33	1.34	1.57	1.85	1.89	1.33	1.38	1.60	1.83	1.89
ϕ	5.48	1.99	8.97	3.42	5.03	6.48	8.05	8.87	3.18	3.78	4.59	5.63	6.74	3.80	4.27	4.91	5.55	6.24
λ	0.71	0.45	0.90	0.62	0.71	0.79	0.84	0.86	0.57	0.62	0.66	0.71	0.76	0.61	0.64	0.68	0.72	0.76
ξ_w	0.73	0.47	0.92	0.58	0.65	0.71	0.78	0.86	0.57	0.62	0.68	0.75	0.82	0.58	0.63	0.69	0.75	0.81
σ_ℓ	1.92	0.18	3.66	0.18	0.18	0.18	0.19	0.54	0.89	1.16	1.55	2.17	2.77	1.07	1.36	1.76	2.24	2.71
ξ_p	0.65	0.40	0.86	0.69	0.74	0.79	0.83	0.85	0.50	0.56	0.63	0.69	0.73	0.52	0.57	0.63	0.68	0.73
ι_w	0.59	0.24	0.89	0.24	0.28	0.38	0.53	0.68	0.32	0.44	0.60	0.76	0.88	0.34	0.47	0.61	0.76	0.88
ι_p	0.22	0.01	0.65	0.04	0.13	0.25	0.38	0.51	0.02	0.08	0.16	0.25	0.34	0.03	0.08	0.16	0.24	0.31
ψ	0.54	0.20	0.86	0.26	0.38	0.59	0.75	0.84	0.38	0.47	0.57	0.69	0.81	0.41	0.48	0.57	0.66	0.77
r_π	2.03	1.45	2.61	1.58	1.78	2.05	2.33	2.55	1.71	1.88	2.10	2.41	2.60	1.76	1.91	2.09	2.35	2.57
ρ	0.81	0.53	0.97	0.74	0.77	0.80	0.82	0.83	0.78	0.80	0.82	0.84	0.85	0.78	0.80	0.82	0.84	0.85
r_y	0.08	-0.04	0.20	0.05	0.07	0.11	0.17	0.20	0.05	0.07	0.08	0.11	0.13	0.06	0.07	0.08	0.10	0.12
$r_{\Delta y}$	0.22	0.10	0.34	0.11	0.15	0.17	0.18	0.19	0.19	0.21	0.22	0.23	0.24	0.20	0.21	0.22	0.23	0.24
ρ_a	0.95	0.00	0.99	0.60	0.93	0.95	0.96	0.96	0.88	0.92	0.94	0.95	0.96	0.90	0.93	0.94	0.96	0.96
ρ_b	0.18	0.00	0.99	0.03	0.08	0.16	0.27	0.75	0.03	0.08	0.14	0.20	0.26	0.04	0.09	0.15	0.21	0.26
ρ_g	0.97	0.00	0.99	0.99	0.99	0.99	0.99	0.99	0.94	0.96	0.97	0.98	0.98	0.94	0.96	0.97	0.98	0.98
ρ_p	0.90	0.00	0.99	0.50	0.66	0.78	0.91	0.97	0.68	0.80	0.87	0.92	0.95	0.74	0.82	0.88	0.92	0.94
ρ_w	0.97	0.00	0.99	0.93	0.97	0.99	0.99	0.99	0.93	0.95	0.97	0.98	0.99	0.94	0.96	0.97	0.98	0.99
μ_p	0.74	0.00	0.99	0.21	0.38	0.64	0.86	0.94	0.31	0.48	0.62	0.73	0.80	0.38	0.51	0.64	0.73	0.80
μ_w	0.88	0.00	0.99	0.72	0.81	0.87	0.91	0.94	0.73	0.80	0.85	0.89	0.92	0.76	0.82	0.86	0.89	0.92
σ_a	0.45	0.00	10.00	0.62	0.70	0.85	1.05	1.22	0.35	0.38	0.42	0.48	0.53	0.37	0.39	0.44	0.48	0.53
σ_b	0.24	0.00	10.00	0.06	0.20	0.24	0.26	0.28	0.21	0.22	0.24	0.26	0.28	0.21	0.23	0.24	0.26	0.28
σ_g	0.52	0.00	10.00	0.51	0.53	0.55	0.57	0.59	0.48	0.49	0.52	0.54	0.56	0.48	0.50	0.52	0.54	0.56
σ_p	0.14	0.00	10.00	0.12	0.14	0.15	0.17	0.18	0.11	0.12	0.14	0.15	0.17	0.11	0.12	0.14	0.15	0.16
σ_w	0.24	0.00	10.00	0.19	0.20	0.22	0.24	0.25	0.21	0.23	0.24	0.26	0.28	0.21	0.23	0.25	0.26	0.28

Notes. The table provides information on the distribution of the indicated parameter across the Monte Carlo replications. See Table 1 for the definitions of the parameters. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a monetary policy disturbance, but the empirical model leaves out the investment disturbance instead.

Table C.7: Parameter estimates across Monte Carlo replications: Second experiment

				misspecified estimation					**ASD procedure**					**SW specification**				
	Truth	**LB**	**UB**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**
α	0.19	0.07	0.31	0.09	0.12	0.16	0.21	0.26	0.14	0.18	0.21	0.25	0.29	0.14	0.17	0.20	0.23	0.26
σ_c	1.39	0.53	2.25	1.39	1.57	2.07	2.24	2.25	1.14	1.27	1.45	1.75	2.14	1.16	1.26	1.41	1.63	1.93
Φ	1.61	1.33	1.89	1.39	1.58	1.79	1.87	1.89	1.33	1.35	1.58	1.85	1.89	1.34	1.43	1.61	1.78	1.87
ϕ	5.48	1.99	8.97	4.09	4.99	6.23	7.43	8.48	3.29	3.85	4.57	5.43	6.54	3.85	4.34	4.98	5.64	6.44
λ	0.71	0.45	0.90	0.54	0.61	0.69	0.76	0.80	0.55	0.61	0.66	0.72	0.77	0.60	0.64	0.68	0.73	0.77
ξ_w	0.73	0.47	0.92	0.55	0.61	0.67	0.72	0.78	0.55	0.60	0.67	0.74	0.81	0.58	0.62	0.69	0.75	0.80
σ_ℓ	1.92	0.18	3.66	0.18	0.18	0.20	0.55	1.03	0.46	0.89	1.56	2.58	3.54	0.67	1.07	1.78	2.64	3.38
ξ_p	0.65	0.40	0.86	0.62	0.68	0.73	0.78	0.83	0.49	0.54	0.62	0.68	0.73	0.51	0.56	0.62	0.68	0.72
ι_w	0.59	0.24	0.89	0.24	0.30	0.42	0.57	0.69	0.32	0.46	0.61	0.78	0.89	0.34	0.47	0.62	0.77	0.88
ι_p	0.22	0.01	0.65	0.04	0.11	0.24	0.34	0.46	0.01	0.07	0.15	0.24	0.32	0.03	0.08	0.16	0.24	0.32
ψ	0.54	0.20	0.86	0.30	0.40	0.53	0.67	0.80	0.37	0.45	0.57	0.70	0.81	0.40	0.48	0.58	0.68	0.79
r_π	2.03	1.45	2.61	1.74	2.04	2.32	2.54	2.60	1.59	1.86	2.28	2.59	2.61	1.68	1.89	2.22	2.52	2.60
ρ	0.81	0.53	0.97	0.79	0.81	0.84	0.86	0.87	0.73	0.77	0.81	0.84	0.86	0.76	0.78	0.81	0.84	0.85
r_y	0.08	-0.04	0.20	0.08	0.10	0.13	0.16	0.19	0.04	0.06	0.10	0.12	0.14	0.05	0.07	0.09	0.12	0.13
$r_{\Delta y}$	0.22	0.10	0.34	0.11	0.13	0.17	0.20	0.22	0.17	0.19	0.23	0.26	0.29	0.17	0.20	0.22	0.25	0.28
ρ_b	0.18	0.00	0.99	0.07	0.14	0.22	0.33	0.53	0.02	0.08	0.14	0.21	0.26	0.04	0.09	0.15	0.21	0.26
ρ_g	0.97	0.00	0.99	0.98	0.99	0.99	0.99	0.99	0.94	0.96	0.97	0.98	0.98	0.94	0.96	0.97	0.98	0.98
ρ_r	0.12	0.00	0.99	0.00	0.00	0.02	0.06	0.10	0.00	0.04	0.11	0.18	0.23	0.01	0.05	0.11	0.17	0.22
ρ_p	0.90	0.00	0.99	0.48	0.65	0.78	0.91	0.97	0.74	0.82	0.89	0.92	0.95	0.78	0.84	0.89	0.92	0.95
ρ_w	0.97	0.00	0.99	0.94	0.96	0.97	0.98	0.99	0.93	0.95	0.97	0.98	0.99	0.94	0.96	0.97	0.98	0.98
μ_p	0.74	0.00	0.99	0.17	0.37	0.61	0.84	0.92	0.36	0.50	0.62	0.72	0.81	0.44	0.55	0.65	0.73	0.80
μ_w	0.88	0.00	0.99	0.73	0.79	0.84	0.89	0.91	0.73	0.79	0.84	0.89	0.91	0.76	0.81	0.85	0.89	0.92
σ_b	0.24	0.00	10.00	0.14	0.19	0.22	0.24	0.26	0.20	0.22	0.24	0.26	0.28	0.21	0.23	0.24	0.26	0.28
σ_g	0.52	0.00	10.00	0.49	0.51	0.53	0.55	0.57	0.47	0.49	0.51	0.54	0.56	0.48	0.50	0.52	0.54	0.56
σ_r	0.24	0.00	10.00	0.22	0.23	0.24	0.25	0.26	0.22	0.23	0.24	0.25	0.26	0.22	0.23	0.24	0.25	0.26
σ_p	0.14	0.00	10.00	0.12	0.14	0.15	0.16	0.18	0.11	0.12	0.14	0.15	0.17	0.11	0.12	0.14	0.15	0.16
σ_w	0.24	0.00	10.00	0.20	0.21	0.23	0.24	0.26	0.21	0.23	0.25	0.27	0.28	0.22	0.23	0.25	0.27	0.28

Notes. The table provides information on the distribution of the indicated parameter across the Monte Carlo replications. See Table 1 for the definitions of the parameters. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a TFP disturbance, but the empirical model leaves out the investment disturbance instead.

Appendix D. Large sample consequences of misspecification

ASDs are designed to deal with the misspecification of structural disturbances. In Section 3, we used Monte Carlo experiments to document that the consequences of such misspecification for parameter estimates can be quite severe.

The analysis of that section has some drawbacks. First, the results are subject to sampling variation, since the sample size was chosen to resemble the length of data series available to macroeconomists in practice. Consequently, the documented deviations from the truth may not be due to misspecification solely but also to small sample issues such as bias and, or course, sampling uncertainty. Second, the analysis only focused on the consequences for parameter estimates whereas it also would be interesting to look at model properties as implied by parameter estimates. Examples are impulse response functions (IRFs) and moments of model variables. Third, we only discussed the results for two representative experiments, whereas there are forty-two possible experiments.

In this appendix, we study the consequences of misspecification in greater detail. First, by using samples of 100,000 observations we reduce sampling variation to negligible levels. Thus, all deviations from the true values are due to misspecification. Second, in addition to parameter estimates we also look at implied moments of model variables and implied IRFs. Third, we consider all possible forty-two experiments. In all other aspects, the experiment is identical to the one described in Section 3.

Appendix D.1. Consequences for parameter values

Table D.8 reports some key percentiles (across experiments) to characterize the range of the estimated parameter values. When constructing percentiles, we only consider parameters that are in both the true and empirical specification.[43] All parameter estimates are affected by misspecification to some extent. Moreover, the minor misspecifications considered in these forty-two experiments lead to massive distortions for several parameters.

The median parameter estimates (across experiments) are relatively close to the true parameter values. Thus, our choice of experiments does not favor bias in a particular direction. There is one exception. The median value of the estimated standard deviation of the productivity disturbance innovation, σ_a, is equal to 0.92 compared to a true value of 0.45. The reason is that this disturbance often "absorbs" the variation of the disturbance that is not included in the empirical specification. Thus, the disturbance that is wrongly included in the empirical specification does not necessarily fulfill this role.

Even if we exclude cases for which the estimates fall in the bottom or top 10%, then we find that estimates are substantially different from their true value for many parameters. For example, for the labor supply elasticity with respect to the real wage, σ_l, the 10^{th} percentile is equal to 0.18 and the 90^{th} percentile is equal to 3.66, compared with a true value of

[43]Specifically, for the parameters of the exogenous random processes, the experiments in which the disturbance is part of the empirical model – but not part of the true *dgp* – are excluded from the calculations of the percentiles.

Table D.8: Parameter values: Point estimates across misspecification experiments

	Truth	Imposed Min	Min	10%	25%	Median	75%	90%	Max	Imposed Max
α	0.19	0.07	0.07	0.11	0.17	0.19	0.20	0.23	0.31	0.31
σ_c	1.39	0.53	0.53	0.78	1.14	1.35	1.60	1.82	2.25	2.25
Φ	1.61	1.33	1.33	1.33	1.53	1.77	1.89	1.89	1.89	1.89
ϕ	5.48	1.99	2.71	3.59	5.47	7.38	8.97	8.97	8.97	8.97
λ	0.71	0.45	0.45	0.59	0.71	0.74	0.84	0.89	0.90	0.90
ξ_w	0.73	0.47	0.50	0.67	0.73	0.75	0.82	0.87	0.91	0.92
σ_ℓ	1.92	0.18	0.18	0.18	0.52	1.87	2.71	3.66	3.66	3.66
ξ_p	0.65	0.40	0.53	0.60	0.65	0.78	0.86	0.86	0.86	0.86
ι_w	0.59	0.24	0.24	0.27	0.38	0.58	0.61	0.80	0.89	0.89
ι_p	0.22	0.01	0.01	0.01	0.10	0.22	0.32	0.48	0.63	0.65
ψ	0.54	0.20	0.20	0.20	0.42	0.54	0.68	0.86	0.86	0.86
r_π	2.03	1.45	1.45	1.45	1.71	2.07	2.39	2.61	2.61	2.61
ρ	0.81	0.53	0.62	0.73	0.79	0.81	0.85	0.88	0.92	0.97
r_y	0.08	-0.04	-0.04	0.01	0.05	0.09	0.16	0.20	0.20	0.20
$r_{\Delta y}$	0.22	0.10	0.10	0.10	0.10	0.20	0.24	0.34	0.34	0.34
ρ_a	0.95	0.00	0.50	0.82	0.92	0.96	0.98	0.99	0.99	0.99
ρ_b	0.18	0.00	0.04	0.09	0.13	0.17	0.26	0.36	0.80	0.99
ρ_g	0.97	0.00	0.94	0.96	0.97	0.97	0.99	0.99	0.99	0.99
ρ_I	0.71	0.00	0.57	0.60	0.68	0.71	0.78	0.84	0.95	0.99
ρ_r	0.12	0.00	0.01	0.06	0.11	0.13	0.18	0.33	0.50	0.99
ρ_p	0.90	0.00	0.70	0.77	0.84	0.89	0.93	0.96	0.98	0.99
ρ_w	0.97	0.00	0.93	0.95	0.97	0.97	0.98	0.99	0.99	0.99
μ_p	0.74	0.00	0.08	0.22	0.43	0.73	0.82	0.91	0.95	0.99
μ_w	0.88	0.00	0.00	0.00	0.87	0.89	0.92	0.96	0.98	0.99
σ_a	0.45	0.00	0.42	0.47	0.67	0.92	1.49	2.57	3.20	10
σ_b	0.24	0.00	0.07	0.20	0.23	0.24	0.26	0.27	0.29	10
σ_g	0.52	0.00	0.52	0.52	0.52	0.53	0.55	0.56	0.57	10
σ_I	0.45	0.00	0.14	0.25	0.39	0.44	0.46	0.48	0.54	10
σ_r	0.24	0.00	0.22	0.23	0.23	0.24	0.26	0.28	0.31	10
σ_p	0.14	0.00	0.04	0.09	0.12	0.14	0.15	0.16	0.17	10
σ_w	0.24	0.00	0.18	0.20	0.21	0.24	0.25	0.29	0.31	10

Notes. This table gives information about the parameter estimates across the forty-two misspecification experiments. For the parameters of the laws of motion of the disturbances, we exclude an experiment from the calculations of the percentiles when the disturbance is part of the empirical model, but not part of the true *dgp*. The table also reports the bounds imposed on parameter estimates. See Table 1 for the definitions of the parameters.

1.92. For the parameter capturing the indexation of wages ι_w, the same two percentiles are 0.27 and 0.80, compared with a true value of 0.59. For the parameter capturing the indexation of prices, ι_p, the two numbers are 0.01 and 0.48, compared with a true value of 0.22. When the two 10% tails are not excluded and the full range of estimates is considered, then the range substantially increases. Specifically, the largest values are 0.89 and 0.63 for the indexation of wages and prices, respectively.[44] Recall that these distortions are solely due to misspecification, not to small-sample variation.

For several parameters, the results remain bad when we narrow the range of outcomes considered. For example, when we exclude the bottom and the top 25%, then the values for σ_l, vary between 0.52 and 2.71 compared with a true value of 1.92. The results are also quite bad for ϕ, the elasticity in the capital adjustment cost function, for which the 25$^{\text{th}}$ percentile is equal to 5.47 and the 75$^{\text{th}}$ percentile is equal to 8.97.

Appendix D.2. Consequences for model properties

The previous section documents that misspecification can lead to large distortions in parameter values. Parameter estimates are often of interest in themselves. At least as important are the properties of the estimated structural model. It could be that different parameter configurations lead to similar model properties. In this section, we address this by looking at implied moments and IRFs.

Appendix D.2.1. Implied model moments

We begin by documenting the consequences of model misspecification for implied model moments using the misspecification setup described above. Table D.9 reports the range of values for typical business cycle properties as implied by the estimated parameter values of the forty-two experiments considered. Specifically, it reports standard deviations and correlation coefficients relative to their true values. Thus, a value equal to 1 means that there is no distortion. The column labeled "true value" reports the range of values the corresponding moment has according to the true *dgp*.[45]

Misspecification implies an upward bias for volatility in our experiments.[46] This upward bias could be specific to our particular type of misspecification. However, the observed upward bias is consistent with the simple analytical example discussed in Appendix D.5.[47]

[44] Parameter estimates are constrained to be in a range, and the largest estimate of the wage indexation parameter is constrained by the imposed upper bound.

[45] Moments are not the same across experiments, since we adjust the standard deviations of the structural disturbances to ensure that the wrongly omitted disturbance does not play an important role.

[46] Section Appendix D.1 documents an upward bias for σ_a, the standard deviation of the TFP disturbance. Since one disturbance is missing from the empirical model, it is not surprising that there is a shift towards some of the other disturbances. By contrast, here we find an upward bias for *total* variability.

[47] In Appendix D.5, we discuss a simple example which documents analytically how maximum likelihood estimation of a misspecified model can lead to an *arbitrarily* large upward bias in the implied variance of an observable.

Table D.9: Moments: Ratio of implied value to truth across experiments with misspecification

	True value (across experiments)	Min	10%	25%	Median	75%	90%	Max
				(estimates, scaled by true value)				
Std(y_t)	[3.48 , 5.12]	0.51	0.78	0.92	1.03	1.64	4.46	6.03
Std(c_t)	[3.30 , 5.58]	0.45	0.76	0.92	1.03	1.81	4.12	6.62
Std(i_t)	[9.73 , 12.94]	0.70	0.87	0.99	1.11	1.71	3.81	6.47
Std(r_t)	[0.52 , 0.61]	0.76	0.90	0.94	1.00	1.36	2.28	2.78
Std(π_t)	[0.37 , 0.54]	0.64	0.72	0.94	1.01	1.25	2.21	2.98
Std(w_t)	[2.13 , 2.70]	0.73	0.83	0.92	1.08	2.28	5.57	10.87
Corr(y_t, c_t)	[0.65 , 0.94]	0.28	0.68	0.93	0.99	1.07	1.15	1.52
Corr(y_t, i_t)	[0.74 , 0.87]	0.69	0.83	0.95	1.00	1.10	1.16	1.29
Corr(c_t, i_t)	[0.63 , 0.89]	-0.68	0.60	0.92	1.00	1.19	1.34	1.57
Corr(c_t, r_t)	[-0.65 , -0.35]	-0.71	0.54	0.86	0.99	1.11	1.52	2.13
Corr(i_t, w_t)	[0.29 , 0.69]	-1.52	0.10	0.64	1.07	1.49	1.99	3.28
Corr(i_t, π_w)	[0.51 , 0.80]	0.36	0.84	0.97	1.02	1.17	1.34	1.75

Notes. This table reports the outcomes across experiments for the indicated moment as implied by parameter estimates relative to its true value. Thus a value equal to 1 indicates that there is no distortion due to misspecification. Each row reports percentiles across our forty-two experiments. It also reports the range of values of the true moments across the experiments. All moments considered are related to variables that are used in the estimation as observables.

The results are solely due to misspecification, since we use very large samples and our ML estimator is consistent when the empirical model is correctly specified.

The overestimation of volatility is enormous in some cases. Even if we exclude the top 25%, then standard deviations can be multiples of the true standard deviation. For example, the 75$^{\text{th}}$ percentile for the standard deviation of wages is 2.28 times its true value. This ratio increases to 5.57 when we only exclude the top 10%. The 90$^{\text{th}}$ percentiles for the consumption and output standard deviation ratios are 4.12 and 4.46, which also indicates massive over-prediction. The 90$^{\text{th}}$ percentile for investment is equal to 3.81 and in the worst experiment the implied standard deviation is 6.47 times as big as the true value. By contrast, the values in the lower tail are less drastic. Excluding the bottom 10%, we find that the largest distortions are found for inflation for which the 10$^{\text{th}}$ percentile is 0.72, that is, implied volatility is 28% below its true value. If we consider all experiments, then the smallest ratio is equal to 0.45, which is found for the implied standard deviation of consumption.

Misspecification also has large quantitative implications for correlation coefficients. In fact, the sign of the correlation coefficient as implied by parameter estimates turns out to be different from its sample analogue in several cases. This would not be a big deal if the two correlation coefficients are both close to zero. But there are also cases in which the implied correlation coefficient according to the estimated empirical model and the true correlation coefficient are both large in absolute value and differ in sign.[48,49]

Appendix D.2.2. Impulse response functions (IRFs)

To conclude the discussion on the consequences of misspecification, we document that misspecification can also have a large impact on impulse response functions. There are many IRFs to consider. Figure D.8 plots for three IRFs the outcomes across the experiments and documents that the distortions can be large. We exclude the cases when the disturbance of interest is in the empirical specification, but not part of the true *dgp*. It would not be surprising if these are different.[50] Thus, the disturbance of interest is part of the true *dgp* as well as the empirical model for all three cases considered.

Figure D.8a plots the response of output to a TFP disturbance. This is obviously a key characteristic of the model. The black line plots the true IRF and the grey lines plot the IRFs as implied by the empirical model for the different experiments. All IRFs are based the same size shock.[51] If the grey lines are close to the black line, then misspecification of the empirical model has only minor consequences for the IRF considered. The sign of the IRF is virtually always correct and TFP disturbances always have a noticeable positive impact on aggregate output.[52] Nevertheless, the figure documents that there are large differences in terms of initial impact, overall magnitude, shape, and persistence.

Figure D.8b plots the response of the real wage to a monetary policy shock. This is clearly the kind of model property one would want to get right when analyzing monetary policy. The figure shows again a wide variety of responses across the different empirical specifications. Whereas the true response is substantial, there are several empirical specifications that predict a very small change. There are also a few specifications that give a much larger response. We want to reemphasize that the plotted IRFs are for a disturbance that is correctly included in the empirical model.

Figure D.8c reports the results for the inflation IRF of an investment-specific shock. For most experiments the IRFs display a similar pattern, but there are important differences in terms of magnitude. For three experiments, however, the IRFs are completely at odds with the true IRF. Whereas the true IRF is positive and has reverted back to zero after twenty periods, the IRFs implied by these three misspecified empirical models are negative and indicate larger volatility and more persistence. Again, relatively small changes in parameter

[48] A striking example is the experiment in which the government disturbance is not present in the true *dgp* and the empirical model excludes the risk-premium disturbance instead. The true correlation between consumption and investment is equal to 0.67 whereas the one implied by the estimated model is equal to -0.41.

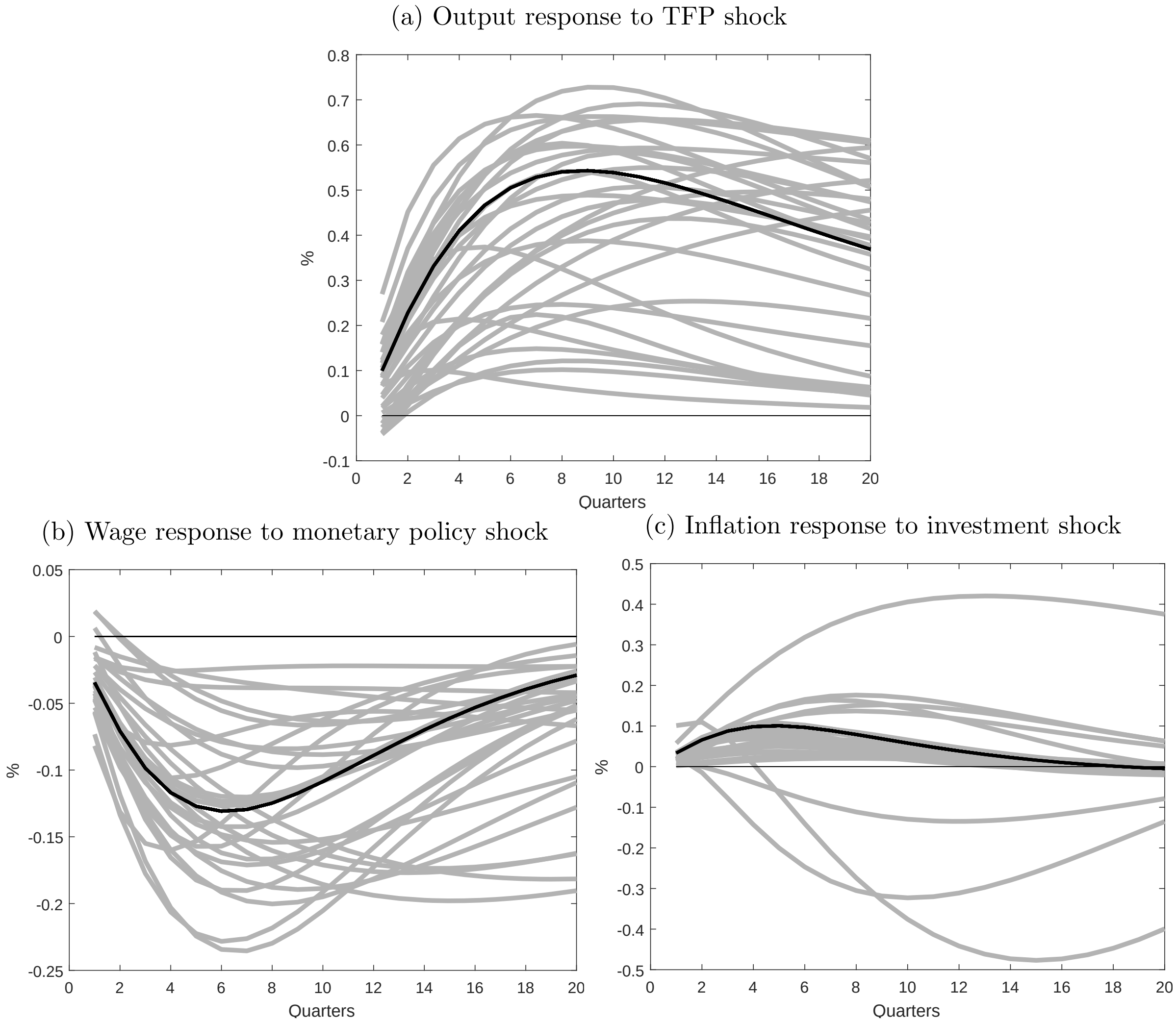
[49] The smallest correlation coefficient (in absolute value) according to the true model is 0.29, so any sign change implies a nontrivial change in the correlation coefficient.

[50] Also, we cannot calculate IRFs for a particular disturbance if that disturbance is not part of the empirical specification. This means that each figure plots IRFs for thirty-two cases.

[51] That is, one standard deviation according to the original SW model. Differences across IRFs are bigger if we use the estimated standard deviations for the different experiments.

[52] In some experiments, the initial response is negative. However, its value is then very small.

Figure D.8: IRFs according to true (black) and misspecified (grey) empirical models



Notes. The figure plots the true IRF (black)and the IRFs implied by the misspecified (grey) empirical models considered. The results are based on a very large sample, so results are not due to small sample variation. These IRFs are for shocks that are correctly included in the model. Also, we do not use estimated standard deviations, but use the same size shock for all IRFs.

values can change these IRFs such that they are much closer to the true IRF.[53]

[53] Specifically, if σ_c, the parameter controlling curvature in the utility function and λ, the parameter indicating the habit component in the utility function, are set equal to their true values, then these three IRFs have a shape that is similar to the true IRF, that is, also predict a hump-shaped positive response. The responses still differ somewhat from the truth in having a more delayed response and a more persistent effect. The estimated values for σ_c in the three experiments are 0.65, 0.53, and 0.53, whereas the true value if equal to 1.39. The estimated values for λ are equal to 0.86, 0.87, and 0.85, whereas the true value is equal to 0.71.

Appendix D.3. Is weak identification the cause?

In Appendix C.2, it was shown that all parameters are identified in all models considered.[54] Moreover, we use a very large sample to estimate the parameters so the large range of values for parameter estimates cannot be caused by samples being too short to be informative. Also, the finding that the different parameter values are associated with quite different model properties indicates that the results discussed in this section are not due to parameters not being identified. As a final check, we compare the values of the likelihood function according to the misspecified model at the estimated values and the true values. When using the true values, we do re-estimate the parameters of the exogenous random variables.[55] The smallest difference between the two log likelihood values is equal to 14.5 and there are only four experiments for which the difference is less than 100. The mean (median) difference is equal to $10,371$ (5501).[56]

Appendix D.4. Choosing Monte Carlo experiments

A careful Monte Carlo experiment requires a sufficiently large number of replications. We use 1,000. Each replication involves a computationally intensive optimization routine. This means we would not be able to do a small-sample version of all 42 experiments in this appendix. The two we use in section 3 were chosen as follows. We ranked all experiments by the likelihood value obtained with the misspecified specification relative to the likelihood value obtained with the correct specification. The idea is that misspecification is less severe if the difference in likelihood values is smaller. The first experiment chosen is the one corresponding to the sixty-sixth percentile and the second is the one corresponding to the thirty-third percentile.[57] Thus, our experiments are neither the least nor the most problematic in terms of misspecification.

Appendix D.5. An analytical example

In this section, we give a *very* simple example to indicate that misspecification can have large distortive effects in the sense that *implied* properties of the model using the parameter estimates can be at odds with the *actual* corresponding properties of the data that are used to

[54]All true specifications have one structural disturbance less than the original SW model. This turns out not to matter for identification. In fact, estimated parameters remain identified when we do the identification test for specifications with five disturbances that exclude the disturbance that is not part of the true *dgp* as well as the one that is erroneously omitted from the empirical specification.

[55]This is a conservative choice, since differences in the likelihoods would be larger if these parameters are not re-estimated.

[56]It is not surprising that across experiments, there are some for which the misspecification is smaller than for others resulting in smaller differences between the two likelihood values. After all, our experiments are not designed to find large misspecification. Our set is constructed using a simple variation in the set of the original structural disturbances.

[57]The first (second) Monte Carlo experiment corresponds to the case when the true *dgp* does not include a monetary policy (TFP) disturbance, but the empirical model leaves out the investment disturbance instead.

estimate the parameters. The model is linear, and all variables have a Normal distribution. Throughout this section, parameter estimates are based on population moments. Thus, the results are not due to small sample variation. The estimation procedure is Maximum Likelihood (ML).

More specifically, this example demonstrates that there can be massive differences between the variances of observables as *implied* by the model using estimated parameter values and the actual variances in the data set. This result is surprising since the ML estimator of the variance of a given time series is the sample variance when the variable has a Normal distribution. We will show that this is not necessarily true for implied variances when the empirical model is misspecified.[58]

True model. The true model is given by the following set of equations:

$$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} = \Lambda \varepsilon_t, \tag{D.1}$$

$$\mathbb{E}\left[\varepsilon_t \varepsilon_t'\right] = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}, \tag{D.2}$$

and we make the following assumption about the distribution of the error terms:

$$\varepsilon_{1,t} \sim N(0, \sigma_1^2) \text{ and } \varepsilon_{2,t} \sim N(0, \sigma_2^2). \tag{D.3}$$

Misspecification. The objective is to estimate the standard deviations of the structural disturbances, σ_1^2 and σ_2^2. The researcher takes the value of Λ as given. The empirical model is misspecified, because $\overline{\Lambda} \neq \Lambda$ is used instead of the true value.

Empirical specifications. We consider the following two empirical specifications:

Case 1: Empirical model given by

$$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \overline{\Lambda} \varepsilon_t, \quad \mathbb{E}\left[\varepsilon_t \varepsilon_t'\right] = \begin{bmatrix} \overline{\sigma}_1^2 & \overline{\sigma}_{12} \\ \overline{\sigma}_{12} & \overline{\sigma}_2^2 \end{bmatrix}. \tag{D.4}$$

[58] As a byproduct of this paper, we learned that there also can be large gaps between *actual* properties of the data used and the corresponding *implied* properties according to the Maximum Likelihood estimates of the model parameters when the DSGE model is correctly specified, but a data sample with finite length is used. Since the objective of Maximum Likelihood is not to match moments, there is no reason why there should be a close match, but we were surprised by the large magnitudes of the differences. For example, using a sample of 1,000 observations generated by the SW model with seven disturbances and the correct empirical specification, it is not unusual to find implied standard deviations for the observables that are three to five times their data counterpart. Such differences will disappear as the sample size increases, since the estimator is consistent, but such asymptotic results do not provide much assurance if there is a small sample bias even at a relatively large sample size of 1,000 observations.

Case 2: Empirical model given by

$$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \overline{\Lambda}\varepsilon_t, \quad \mathbb{E}\left[\varepsilon_t \varepsilon_t'\right] = \begin{bmatrix} \overline{\sigma}_1^2 & 0 \\ 0 & \overline{\sigma}_2^2 \end{bmatrix}. \tag{D.5}$$

Both empirical models are misspecified, because they use the wrong value of Λ. In the first case, the empirical model allows the correlation between the two innovations to be non-zero even though it is equal to zero according to the true data generating process. In the second case, the empirical model imposes that the correlation is equal to zero, just as it is in the true model.

Case 1: Wrong Λ and allow for wrong σ_{12}. Since the model is linear and the shocks have a normal distribution, the ML estimator of the variance-covariance matrix $\mathbb{E}\left[\varepsilon_t \varepsilon_t'\right]$, $\widehat{\Sigma}_\varepsilon$, is given by

$$\widehat{\Sigma}_\varepsilon \;=\; \overline{\Lambda}^{-1}\widehat{\Sigma}_y'\overline{\Lambda}^{-1\prime}. \tag{D.6}$$

As mentioned above, we abstract from sampling variation and $\widehat{\Sigma}_y'$ is estimated using population moments. This means that the ML estimator of $\widehat{\Sigma}_\varepsilon'$ is given by

$$\begin{aligned} \widehat{\Sigma}_\varepsilon &= \overline{\Lambda}^{-1}\mathbb{E}\left[y_t y_t'\right]\overline{\Lambda}^{-1\prime} && \text{(D.7)} \\ &= \overline{\Lambda}^{-1}\Lambda\Lambda'\overline{\Lambda}^{-1\prime}. && \text{(D.8)} \end{aligned}$$

True versus implied variance. The purpose of this section is to document the consequences of misspecification for the implied variance of the observable y_t according to the estimated model. The *true* variance-covariance matrix is given by:

$$\Sigma_y^{\text{true}} = \mathbb{E}\left[y_t y_t'\right] = \Lambda\Lambda'. \tag{D.9}$$

The *implied* variance of y_t according the researcher's (misspecified) model, $\widehat{\Sigma}_y$, is given by

$$\begin{aligned} \widehat{\Sigma}_y &= \overline{\Lambda}\widehat{\Sigma}_\varepsilon\overline{\Lambda}' && \text{(D.10)} \\ &= \overline{\Lambda}\overline{\Lambda}^{-1}\Lambda\Lambda'\overline{\Lambda}^{-1\prime}\overline{\Lambda}' && \text{(D.11)} \\ &= \Lambda\Lambda' = \Sigma_y^{\text{true}}. && \text{(D.12)} \end{aligned}$$

Thus, the procedure actually generates the correct answer even though an incorrect empirical specification is used. In this case, the estimated empirical model is misspecified for two reasons, namely it has the wrong Λ and the estimated value of σ_{12} is not equal to its true value. These have exactly offsetting effects in terms of their impact on the implied variance. Another way to look at this result is the following. By allowing for a more flexible specification, i.e., a non-zero value for σ_{12}, the researcher would get a better answer for the implied variance of y_t even though the flexibility implies that the estimated model is wrong in more dimensions.

Case 2: Wrong Λ and correct σ_{12}. Obtaining the estimate for $\widehat{\Sigma}_\varepsilon$ is just as easy as in the previous case. Given $\widehat{\Lambda}$ and data for y_t, one can calculate the values for ε_t and use these to calculate the variance of ε_t and the implied variance of y_t. The following is a complicated, but useful way to express the outcome:

$$\widehat{\Sigma}_\varepsilon = \begin{array}{c} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ + \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \end{array} \tag{D.13}$$

True versus implied variance. The implied variance of y_t is equal to

$$\widehat{\Sigma}_y = \left(\begin{array}{c} \overline{\Lambda} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}' \\ + \\ \overline{\Lambda} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \overline{\Lambda}' \end{array} \right) \neq \Lambda\Lambda' = \Sigma_y^{\text{true}} \tag{D.14}$$

The reason $\widehat{\Sigma}_y \neq \Sigma_y^{\text{true}}$ is that the $\overline{\Lambda}$ terms do not cancel out. In our Monte Carlo experiments with misspecified models, we find that there often are large gaps between the variances of the observables used in the estimation and the corresponding variances as implied by the model using the estimated parameters. Moreover, there is a bias. That is, the implied variance is typically larger than the actual variance. Our Monte Carlo experiments are a lot more complicated than this example, but this example may shed light on the coincidence of high implied variances. Specifically, because the $\overline{\Lambda}$s do not cancel out, the expression for $\widehat{\Sigma}_y$ contains terms like the following:

$$\overline{\Lambda} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}^{-1} = \frac{1}{\overline{\lambda}_{11}\overline{\lambda}_{22} - \overline{\lambda}_{12}\overline{\lambda}_{21}} \begin{bmatrix} \overline{\lambda}_{11}\overline{\lambda}_{22} & -\overline{\lambda}_{11}\overline{\lambda}_{12} \\ \overline{\lambda}_{21}\overline{\lambda}_{22} & -\overline{\lambda}_{12}\overline{\lambda}_{21} \end{bmatrix}. \tag{D.15}$$

This equation documents that the ratio of the implied variance relative to the true variance could be arbitrarily large if the term in the denominator goes to zero.[59] For a correctly specified model this would not matter, since the small term in the denominator would then be offset by an equally small term in the numerator. But this is not necessarily the case for an incorrectly specified model.

[59] The opposite is less likely, since it would require values for the λ_{ij} coefficients such that the combinations appearing in square brackets are small, but the particular combination in the denominator is not. For example, one cannot accomplish this by simply choosing small values for the λ_{ij} terms.

Appendix E. ASD procedure for the Smets-Wouters model

In this appendix, we provide further details on how the ASD procedure is implemented in Section 4 and we provide additional results.

Appendix E.1. Including ASDs in SW equations

To apply the ASD procedure to the SW model, we adapt the Dynare program provided by the authors.[60] Adapting a Dynare program to add an agnostic disturbance is easy. Specifically, for the first ASD, $\widetilde{\varepsilon}_{A,t}$, we do the following.

1. In the model block, we add $\widetilde{\Upsilon}_{j,A}\widetilde{\varepsilon}_{A,t}$ to the j^{th} equation, where $\widetilde{\varepsilon}_{A,t}$ is the agnostic disturbance and $\widetilde{\Upsilon}_{j,A}$ the coefficient associated with the agnostic disturbance in the j^{th} equation. Details are given below.[61]

2. We add an equation to the model block that describes the law of motion for $\widetilde{\varepsilon}_{A,t}$. If the agnostic disturbance replaces a regular structural disturbance, then this disturbance should be taken out of the program.

3. Declare $\widetilde{\varepsilon}_{A,t}$ as a variable and declare the elements of $\widetilde{\Upsilon}_{j,A}$ and the coefficients of the law of motion for $\widetilde{\varepsilon}_{A,t}$ as parameters.

4. Specify a prior for the elements of $\widetilde{\Upsilon}_{j,A}$.

We do not add the agnostic disturbance to Equations (6) and (12) of the SW model, because these equations just contain definitions for capacity utilization and the wage mark-up, respectively.[62] The set of equations for the SW model consists of two parts. The first part models the flexible price economy and the second part models the actual economy with sticky prices. One needs to model the flexible-price economy, because the flexible-price output level is used to define the output gap, which is one of the arguments in the monetary policy rule. In principle, one could let the agnostic disturbance enter the equations of the sticky-price economy and the associated equations in the flexible-price economy with a different coefficient.[63] Given the minor role played by the flexible-price block, it doesn't quite make sense to introduce so many additional parameters. Moreover, structural disturbances would enter the associated pair of equations in the same way in most economic models. Therefore, we also restrict the agnostic disturbance to enter the associated equations in the same way.

[60] The program is available at https://www.aeaweb.org/articles?id=10.1257/aer.97.3.586 under the "Download Data Set" link.

[61] The other two ASDs are added using the same procedure.

[62] Equation numbers refer to those in Smets and Wouters (2007). We do allow the agnostic disturbances to affect the utilization rate and the wage mark-up directly by including it in the model equations that specify their relationship with other model variables.

[63] The sticky-price block contains some equations, such as the monetary policy rule, that do not have a counterpart in the flexible-price economy.

The exception is SW Equation (13) because it captures both potential stickiness in wages and the relationship between the wage rate and its mark-up.

Specifically, we add the agnostic disturbance to Equations (1), (2), (3), (4), (5), (7), (8), (9), and (11) of the SW model and the associated equations of the flexible-price economy. We also add it to Equation (13) in both the flexible and the sticky-price part of the model, but here we allow coefficients to differ. In addition, we add the agnostic disturbance to Equations (10) and (14) which do not have a counterpart in the flexible-price economy. This means that $\widetilde{\Upsilon}_{\cdot,A}$ has thirteen elements. The last coefficient associated with the agnostic disturbance is the autoregressive coefficient of its law of motion. The standard deviation of the agnostic disturbance is normalized to be equal to 1.

Additional information. The SW specification uses consumption growth as an observable and has an equation that defines consumption growth. Allowing an agnostic disturbance to affect this equation would capture measurement error (which would be correlated with structural disturbances if this ASD also appears in other model equations with a non-zero coefficient). We do not explore this possibility to keep the analysis parsimonious and to stay close the SW approach, which does not allow for measurement error.

As pointed out in the main text, the prior mean of $\widetilde{\varepsilon}_{A,t}$ and $\widetilde{\varepsilon}_{B,t}$ are set equal to associated values of $\varepsilon_{b,t}$ and $\varepsilon_{i,t}$. For example, suppose we use the ASD procedure to test the restrictions of the risk-premium disturbance by replacing it with an ASD. The risk-premium disturbance appears in two equations, namely the consumption/bond Euler equation and the capital-valuation equation. The prior means of the reduced-form agnostic coefficients for these two equations are set equal to the values according to the SW restrictions with structural parameters evaluated at their prior means. The reduced-form coefficients associated with the other equations have a prior mean equal to zero. Having a non-zero prior has a practical advantage. The signs of the coefficients of an agnostic disturbance are not identified. That is, one can switch the signs of the coefficients of an ASD as long as one does it for all coefficients. A necessary consequence of its agnostic nature is that the sign of an ASD has no a priori meaning. If the prior means of all ASD coefficients are zero, then the ASD coefficients can flip sign for different runs of the MCMC procedure.

Appendix E.2. Model selection procedures

Which structural disturbances to include? The first stage of the model selection procedure is to decide which regular and agnostic structural disturbances to include. Specifically, we compare a set of models that do or do not include the risk-premium disturbance, that do or do not include the investment disturbance, and that include one, two, or three ASDs.[64] We still allow the risk-premium and the investment-specific disturbance to appear in the final set even though replacement by an ASD improved model fit. The reason is that a specification

[64] To estimate the model with all seven observables, an empirical specification with only one ASD would need either the risk-premium or the investment disturbance to avoid a singularity.

with both ASDs and these regular disturbances could perform even better.

Table E.10: Model selection procedure for SW model: Step 1

regular structural		agnostic			marginal
$\varepsilon_{b,t}$	$\varepsilon_{i,t}$	$\widetilde{\varepsilon}_{A,t}$	$\widetilde{\varepsilon}_{B,t}$	$\widetilde{\varepsilon}_{C,t}$	data density
no	no	yes	yes	no	**-906.85**
no	no	yes	yes	yes	-925.55
no	yes	yes	no	no	-908.79
no	yes	yes	yes	no	-907.46
no	yes	yes	yes	yes	-922.94
yes	no	no	yes	no	-919.81
yes	no	yes	yes	no	-907.32
yes	no	yes	yes	yes	-921.71
yes	yes	no	no	no	-922.40
yes	yes	yes	no	no	-909.35
yes	yes	no	yes	no	-920.26
yes	yes	yes	yes	no	-908.09
yes	yes	yes	yes	yes	-922.82

Notes. The table reports the marginal data density for different empirical specifications regarding three agnostic disturbances and the two disturbances that are misspecified, that is, the risk-premium disturbance, $\varepsilon_{b,t}$, and the investment disturbance, $\varepsilon_{i,t}$. The number in bold indicates the highest outcome.

Table E.10 reports the results. It shows that the model with the highest marginal data density is one with two agnostic disturbances, without the SW risk-premium, and without the SW investment-specific disturbance. Another indication that there is no need for these two SW structural disturbances is that their role in terms of explaining variation in the data is very small when agnostic disturbances are included. According to the (unconditional) variance decomposition of the estimated SW model, the risk-premium disturbance is especially important for the price of capital, consumption growth, and output growth explaining 45.4%, 61.2%, and 22.1% of total variability, respectively. It only plays a minor role for other variables. When agnostic disturbances are added, then these three numbers drop to 3.88%, 3.88%, and 2.05%, respectively.[65] The reduction in the role of the investment disturbance is even stronger. In the SW model, the investment disturbance plays a quantitatively important role for many variables. For investment growth it even explains 82.1% of the volatility. With agnostic disturbances added, its role becomes minuscule. Even for investment growth it only explains 0.31%.

Obtaining a concise ASD specification. To interpret ASDs, we could use the best specification found so far. However, interpretation of an ASD is easier when the specification is more concise. To determine whether an agnostic disturbance should be excluded from

[65]These numbers are based on the specification with two ASDs and all seven SW structural disturbances using posterior mode estimates.

some equations, we implement model selection procedures using the marginal data density as the criterion of fit. This statistic increases when fit improves, but also penalizes additional parameters.

We consider both a specific-to-general procedure and a general-to-specific procedure and we apply the procedure for the specifications with two and three ASDs.[66] The details of these procedures are described further below. The specific-to-general procedure with three ASDs leads to the highest MDD and the selected outcome will be our preferred empirical model. The specific-to-general procedure with two ASDs and the general-to-specific procedure with two ASDs lead to slightly lower MDDs.[67] Moreover, the models selected by these three procedures are very similar. Specifically, the additional ASD in the specification with three ASDs only plays a minor role. The zero restrictions imposed for the other two ASDs are not exactly the same, but the differences are due to coefficients that turn out to be small. As documented in Appendix E.3, the estimates of the parameters are similar and the estimates obtained with these three empirical specifications imply similar model properties. The general-to-specific procedure with three ASDs leads to a specification that has a much lower MDD.[68]

In our preferred specification, the first agnostic disturbance enters eight of the thirteen equations, the second in three, and the third in five. By contrast, the original SW risk-premium and the investment-specific disturbance appear in only two.

Details of the model selection procedures. The general-to-specific model selection procedure starts with the specification in which the agnostic disturbances are allowed to enter each model equation. It then calculates the marginal data densities for all possible specifications in which the ASD is *not* allowed to enter *one* of the model equations. Thus, we estimate a set of models, each having one less coefficient. If none of the specifications lead to a better fit, then the procedure stops. If improvements are found, then the procedure is repeated using the specification that led to the biggest improvement as the benchmark.

The specific-to-general procedure starts with the specifications in which each of the two ASDs are allowed to enter only one model equation. To avoid a singularity, one cannot

[66]An informal alternative selection procedure would be the following. One starts at the same point as the general-to-specific procedure, that is, with ASDs included in every equation. The marginal posteriors of the agnostic coefficients provides information on the lack of importance of different agnostic coefficients and may provide the researcher promising combinations of zero restrictions to impose. In fact, the posteriors for the coefficients with the fully unrestricted ASD specifications are very predictive of the equations selected by the specific-to-general procedures for this application. Of course, there are good reasons why this informal procedure is not a generally accepted model selection procedure and we cannot expect this to always work well.

[67]The specific-to-general procedure generates an MDD equal to -892.92 with two ASDs and -890.76 with three. The general-to-specific with two ASDs results in an MDD of -894.94.

[68]Namely, -909.48. The general-to-specific procedure already stops after two steps. That is, the procedure does not detect that imposing *multiple* restrictions *simultaneously* does lead to substantial improvements. One has to impose some structure on any model selection procedure, because it would be impossible to consider all possible combinations. That is, one has to give instructions on what paths to follow and which ones to ignore. But this means that the model selection procedure may not find the best model. This motivates our use of different model selection criteria.

start with a more parsimonious model.[69] In the next step, we estimate a set of models in which one of the ASDs is added to one equation and, thus, one additional parameter is estimated. The procedure stops if none of the specifications leads to an improvement. If there is an improvement, then the specification with the largest improvement becomes the next benchmark and the procedure is repeated.

Why not consider even more general specifications? Although our model selection procedures consider a rich set of models, they are not the most general. Unfortunately, there are practical limitations to what is feasible. Five SW disturbance are always included in our specifications. The most ideal setup would be flexible in this dimension as well and not safeguard any of the seven SW regular disturbances and allow for the possibility of including seven ASDs (or more). With such a setup all SW disturbances could be replaced by an ASD. The first problem one would have to deal with is that identification of structural parameters is likely to limit the number of regular structural disturbances one can replace with ASDs. Let us consider a simple setup in which there are seven equations for seven state variables and all state variables are observables. Moreover, each equation has one regular structural disturbance. A general-to-specific procedure would be complicated since the first-stage model would have a large number of coefficients to estimate. Specifically, if all seven ASDs appear in all equations, then one needs to estimate forty-nine reduced-form coefficients. One may need a rich data set to identify all of them. In our application, the number of coefficients would be equal to ninety-one, since we have thirteen equations. The specific-to-general procedure faces the problem that each specification needs at least seven disturbances to avoid singularities. This means that there are a large number of different models one can start with. For the simple setup with seven equations described above, this would mean that there are already $2^7 = 128$ different models to consider in the first round alone.

Different prior for ASD coefficients. When we narrow the prior of the agnostic coefficients by reducing the standard deviation to 0.1, then the restrictions of the monetary policy disturbance are also rejected. But the increase in the marginal data density is relatively small, namely from -922.40 to -920.82. The less informed prior of the main text is more consistent with the idea of the ASDs being agnostic disturbances.

Appendix E.3. Additional results

Correlation of the estimated innovations. Tables E.11 and E.12 report the contemporaneous correlation coefficients of the estimated innovations for the ASD and SW specification, respectively. We use the posterior mean estimates to construct the smoothed shocks. For the SW specification with seven innovations, nine correlation coefficients are significantly

[69] The posteriors of the ASD coefficients in the fully agnostic model provide clear evidence that one of the ASDs is very important for the bond Euler equation and one for the investment Euler equation. So these are natural choices.

different from zero at the 10% or lower level. For the ASD specification with eight innovations, four coefficients are significant and only two when we exclude the eighth innovation associated with $\widetilde{\varepsilon}_{C,t}$.

Table E.11: Cross-correlation of innovations: SW

	η_a	η_g	η_r	η_p	η_w	η_b	η_i
η_a	.	0.010	-0.024	-0.135*	0.150	0.116	-0.260**
η_g	.	.	0.166**	0.218**	-0.160**	-0.262**	-0.074
η_r	.	.	.	-0.056	-0.048	0.186*	0.037
η_p	.	.	.	.	-0.098	-0.221**	0.070
η_w	.	.	.	.	.	-0.019	-0.189**
η_b	.	.	.	.	.	.	0.152

Notes. * (**) indicates significant at the 10% (5%) level. Standard errors are calculated using the VARHAC estimator of Den Haan and Levin (1997) which corrects for serial correlation.

Table E.12: Cross-correlation of innovations: ASD

	$\widetilde{\eta}_a$	$\widetilde{\eta}_g$	$\widetilde{\eta}_r$	$\widetilde{\eta}_p$	$\widetilde{\eta}_w$	$\widetilde{\eta}_A$	$\widetilde{\eta}_B$	$\widetilde{\eta}_C$
$\widetilde{\eta}_a$	.	0.027	0.010	0.006	0.103	0.009	-0.088	0.066
$\widetilde{\eta}_g$	.	.	0.150	0.073	0.189**	0.041	-0.049	-0.116
$\widetilde{\eta}_r$	.	.	.	-0.054	-0.014	-0.029	-0.043	-0.047
$\widetilde{\eta}_p$	.	.	.	.	0.260**	-0.021	0.011	-0.176**
$\widetilde{\eta}_w$	.	.	.	.	.	-0.051	0.038	0.629**
$\widetilde{\eta}_A$	.	.	.	.	.	.	0.062	0.003
$\widetilde{\eta}_B$	.	.	.	.	.	.	.	-0.153

Notes. * (**) indicates significant at the 10% (5%) level. Standard errors are calculated using the VARHAC estimator of Den Haan and Levin (1997) which corrects for serial correlation.

Table E.13: Auto-correlation of innovations

	ASD		SW
$\widetilde{\eta}_a$	-0.060	η_a	-0.040
$\widetilde{\eta}_g$	-0.024	η_g	-0.182**
$\widetilde{\eta}_r$	-0.170	η_r	-0.013
$\widetilde{\eta}_p$	-0.121	η_p	-0.077*
$\widetilde{\eta}_w$	0.069	η_w	-0.043
$\widetilde{\eta}_A$	-0.069	η_b	-0.071*
$\widetilde{\eta}_B$	-0.155**	η_i	-0.148**
$\widetilde{\eta}_C$	-0.245**		

Notes. * (**) indicates significant at the 10% (5%) level. Standard errors are calculated using the VARHAC estimator of Den Haan and Levin (1997) which corrects for serial correlation.

Table E.13 reports the auto-correlation coefficients for both empirical specifications. Again the ASD specification does quite a bit better with only two significant coefficients (at the 10% level) for its eight innovations compared to four of the seven for the SW specification.

Impact on parameter estimates and model properties. Table E.14 documents there are several differences between the estimated values of the structural parameters obtained with the fully structural SW specification and our preferred agnostic specification with three ASDs. For example, the inflation coefficient in the Taylor rule is equal to 2.05 in the SW specification and 1.77 in ours.[70] The SW estimate is right at the upper bound of our 90% highest posterior density (HPD) interval. The SW mean estimate for the parameter characterizing the share of fixed cost in production is equal to 1.61 which is quite a bit higher than our mean estimate of 1.47 and outside our 90% HPD interval. Also, the mean posterior value of the MA coefficient of the wage mark-up disturbance is equal to 0.85 according to the SW specification and 0.59 according to ours. Our mean estimate for the standard deviation of this disturbance is roughly a third of the SW estimate.

Although there are some nontrivial differences, they are relatively small and the IRFs of the five regular structural disturbances that are included in both specifications are very similar for the two empirical models. The same is true when we consider the role of these five disturbances for the variance decomposition. Details are given in Tables E.15 and E.16. One nontrivial change is the role of the productivity disturbance for output growth, which is 16.1% according to SW and 22.2% according to ours. Although the differences seem minor if we consider the five structural disturbances in isolation, the combined role changes quite a bit for some variables. For example, the combined role of these five structural disturbances for investment (amount of capital used) is equal to 55.5% (74.1%) for the SW specification and 68.7% (92.6%) for our preferred specification.

[70]We report posterior mean estimates unless indicated otherwise.

Table E.14: Posterior Means

Parameter	Original SW	Agnostic: 2 ASDs concise	Agnostic: 2 ASDs unrestricted	Agnostic: 3 ASDs concise	Agnostic: 3 ASDs unrestricted
α	0.1903	0.2044	0.1878	0.1877	0.2089
σ_c	1.3889	1.4657	1.4535	1.4618	1.4772
Φ	1.6083	1.5211	1.5242	1.4741	1.4762
ϕ	5.7405	5.3843	4.4031	5.3425	4.6933
λ	0.7136	0.6544	0.7055	0.6679	0.6930
ξ_w	0.7066	0.6660	0.6706	0.7268	0.6453
σ_ℓ	1.8458	1.9094	1.7733	2.0770	1.5916
ξ_p	0.6541	0.6566	0.6981	0.6412	0.6902
ι_w	0.5783	0.5556	0.5432	0.5077	0.5557
ι_p	0.2389	0.2010	0.1997	0.1871	0.1891
ψ	0.5426	0.5345	0.5049	0.5283	0.3176
r_π	2.0469	1.7676	1.7797	1.7746	1.7438
ρ	0.8105	0.7933	0.8082	0.8018	0.8032
r_y	0.0887	0.0725	0.0860	0.0787	0.0819
$r_{\Delta y}$	0.2237	0.1903	0.1703	0.1941	0.1608
ρ_a	0.9572	0.9555	0.9483	0.9532	0.9510
ρ_g	0.9764	0.9719	0.9710	0.9702	0.9018
ρ_r	0.1464	0.1376	0.1219	0.1286	0.1227
ρ_p	0.8893	0.8975	0.8899	0.9262	0.9080
ρ_w	0.9680	0.9751	0.9790	0.9747	0.9822
ρ_b / ρ_A	0.2165	0.3344	0.6386	0.3239	0.4527
ρ_i / ρ_B	0.7116	0.6087	0.1660	0.6069	0.7232
ρ_C	-	-	-	0.1865	0.1577
μ_p	0.6977	0.6764	0.6923	0.7166	0.7172
μ_w	0.8466	0.8241	0.8368	0.5945	0.8168
ρ_{ga}	0.5184	0.6438	0.6525	0.6709	0.5448
σ_a	0.4586	0.4436	0.4421	0.4524	0.4411
σ_g	0.5299	0.4702	0.4689	0.4428	0.2285
σ_r	0.2449	0.2180	0.2171	0.2171	0.2114
σ_p	0.1403	0.1346	0.1299	0.1308	0.1311
σ_w	0.2427	0.2384	0.2361	0.0763	0.2249
σ_b	0.2398	-	-	-	-
σ_i	0.4525	-	-	-	-
$100(\beta^{-1}-1)$	0.1648	0.1685	0.1826	0.1656	0.2038
$\bar{\gamma}$	0.4316	0.4349	0.4386	0.4367	0.4352
$\bar{\pi}$	0.7845	0.7483	0.7443	0.7391	0.7534
$\bar{\ell}$	0.5617	0.1263	0.5216	0.1303	1.0360
MDD	-922.40	-892.92	-906.85	-890.73	-925.50

Notes. MDD stands for marginal data density. The "concise" ASD specifications are the ones chosen by the specific-to-general model selection procedure. The "unrestricted" ASD specifications are the fully agnostic with no zero restrictions. See Table 1 for the definitions of the parameters.

Table E.15: Variance decomposition for observables across model specifications

		ε_a	ε_g	ε_r	ε_p	ε_w	$\varepsilon_b/\widetilde{\varepsilon}_A$	$\varepsilon_i/\widetilde{\varepsilon}_B$	$\widetilde{\varepsilon}_C$
Δy	Original SW	16.10	28.88	6.17	4.55	6.39	22.12	15.79	-
	Agnostic: 2 ASDs	20.29	27.01	7.15	6.04	8.12	20.53	10.85	-
	Agnostic: 3 ASDs	22.21	24.60	7.04	4.66	10.30	21.33	8.04	1.82
Δc	Original SW	5.29	2.10	11.56	4.40	14.54	61.17	0.95	-
	Agnostic: 2 ASDs	3.26	1.62	11.29	4.56	15.33	62.34	1.61	-
	Agnostic: 3 ASDs	2.95	1.28	10.69	3.37	17.90	61.67	2.03	0.1
Δi	Original SW	6.01	0.84	2.47	3.80	2.37	2.46	82.05	-
	Agnostic: 2 ASDs	4.86	0.91	2.19	4.24	2.76	12.25	72.80	-
	Agnostic: 3 ASDs	5.49	1.02	2.38	3.80	3.94	12.55	70.01	0.81
ℓ	Original SW	1.94	10.34	3.15	6.23	67.66	2.52	8.15	-
	Agnostic: 2 ASDs	1.29	6.84	2.47	6.04	71.23	1.56	10.57	-
	Agnostic: 3 ASDs	1.08	4.33	2.15	4.44	79.70	1.29	4.97	2.03
Δw	Original SW	4.53	0.09	1.48	29.47	61.61	0.79	2.03	-
	Agnostic: 2 ASDs	3.82	0.22	2.43	30.84	54.34	3.02	5.34	-
	Agnostic: 3 ASDs	4.09	0.11	1.25	25.18	13.32	2.23	0.38	53.45
π	Original SW	3.92	1.00	4.25	27.64	59.43	0.58	3.18	-
	Agnostic: 2 ASDs	3.16	1.28	4.43	24.91	61.96	0.79	3.46	-
	Agnostic: 3 ASDs	2.95	0.90	3.28	16.87	70.46	0.68	3.96	0.91
r	Original SW	10.09	3.90	14.67	7.17	38.42	7.40	18.34	-
	Agnostic: 2 ASDs	6.50	3.49	9.77	5.79	38.96	21.49	14.02	-
	Agnostic: 3 ASDs	5.70	2.77	8.18	4.33	48.61	17.29	12.47	0.65

Notes. The table provides the contributions (in percent) of the different structural disturbances to the variance of the observable variables, across different model specifications. The ASD specifications are the ones chosen by our model selection procedure. y stands for log output; c for log consumption; i for log investment; l for hours; w for log wage rate; π for inflation; and r for nominal interest rate. Structural disturbances are defined as follows. ε_a: TFP; ε_g: government expenditures; ε_r: monetary policy; ε_p:price mark-up; ε_w: wage mark-up; ε_b: risk premium; ε_i: investment; $\widetilde{\varepsilon}_A$: agnostic Euler; $\widetilde{\varepsilon}_B$: agnostic investment-modernization; and $\widetilde{\varepsilon}_C$: capital-efficiency wage mark-up.

Table E.16: Variance decomposition for additional variables across model specifications

		ε_a	ε_g	ε_r	ε_p	ε_w	$\varepsilon_b/\widetilde{\varepsilon}_A$	$\varepsilon_i/\widetilde{\varepsilon}_B$	$\widetilde{\varepsilon}_C$
y_t	Original SW	29.93	4.09	2.16	6.37	48.58	1.53	7.34	-
	Agnostic: 2 ASDs	26.50	3.02	1.91	7.02	55.93	1.31	4.32	-
	Agnostic: 3 ASDs	21.19	2.13	1.67	5.47	65.95	1.14	2.17	0.28
c_t	Original SW	11.06	8.42	2.08	4.19	69.25	2.18	2.83	-
	Agnostic: 2 ASDs	6.60	6.60	1.78	4.23	78.76	1.81	0.22	-
	Agnostic: 3 ASDs	4.29	4.30	1.52	3.16	84.48	1.51	0.49	0.25
i_t	Original SW	20.37	5.41	1.27	6.93	21.56	0.22	44.23	-
	Agnostic: 2 ASDs	17.22	6.35	1.14	8.25	29.78	1.21	36.04	-
	Agnostic: 3 ASDs	15.31	5.79	1.13	7.75	38.72	1.06	29.25	1.00
r_t^k	Original SW	14.86	17.47	1.63	10.58	19.21	0.86	35.39	-
	Agnostic: 2 ASDs	12.28	20.44	2.65	19.09	29.73	0.92	14.88	-
	Agnostic: 3 ASDs	8.41	14.66	1.73	13.16	30.17	0.67	18.12	13.08
q_t	Original SW	4.65	0.55	9.03	3.11	1.20	45.42	36.04	-
	Agnostic: 2 ASDs	9.78	1.35	21.83	9.30	3.67	19.58	34.49	-
	Agnostic: 3 ASDs	9.72	1.35	19.88	6.50	5.14	18.64	31.56	7.21
z_t	Original SW	14.86	17.47	1.63	10.58	19.21	0.86	35.39	-
	Agnostic: 2 ASDs	12.23	20.36	2.64	19.02	29.61	4.43	11.71	-
	Agnostic: 3 ASDs	8.86	15.43	1.82	13.85	31.76	4.14	9.46	14.68
μ_t^p	Original SW	11.56	0.29	3.27	57.02	23.87	0.87	3.11	-
	Agnostic: 2 ASDs	8.06	0.37	3.38	53.22	18.90	14.59	1.48	-
	Agnostic: 3 ASDs	7.99	0.24	2.13	54.80	11.88	15.22	2.61	5.13
k_t^s	Original SW	23.43	3.92	1.23	11.37	34.19	0.36	25.50	-
	Agnostic: 2 ASDs	21.82	4.90	1.55	16.51	52.66	1.32	1.24	-
	Agnostic: 3 ASDs	15.59	3.60	1.11	14.11	58.20	1.21	0.61	5.57
k_t	Original SW	22.38	8.11	0.50	4.93	31.56	0.04	32.48	-
	Agnostic: 2 ASDs	22.16	11.30	0.55	7.27	55.74	0.21	2.77	-
	Agnostic: 3 ASDs	14.84	8.05	0.42	6.18	58.26	0.12	2.37	9.75
w_t	Original SW	33.03	1.03	1.95	38.38	18.61	0.40	6.60	-
	Agnostic: 2 ASDs	26.99	1.00	2.63	47.34	20.71	0.39	0.92	-
	Agnostic: 3 ASDs	25.35	0.74	1.62	49.34	14.29	0.30	0.44	7.92

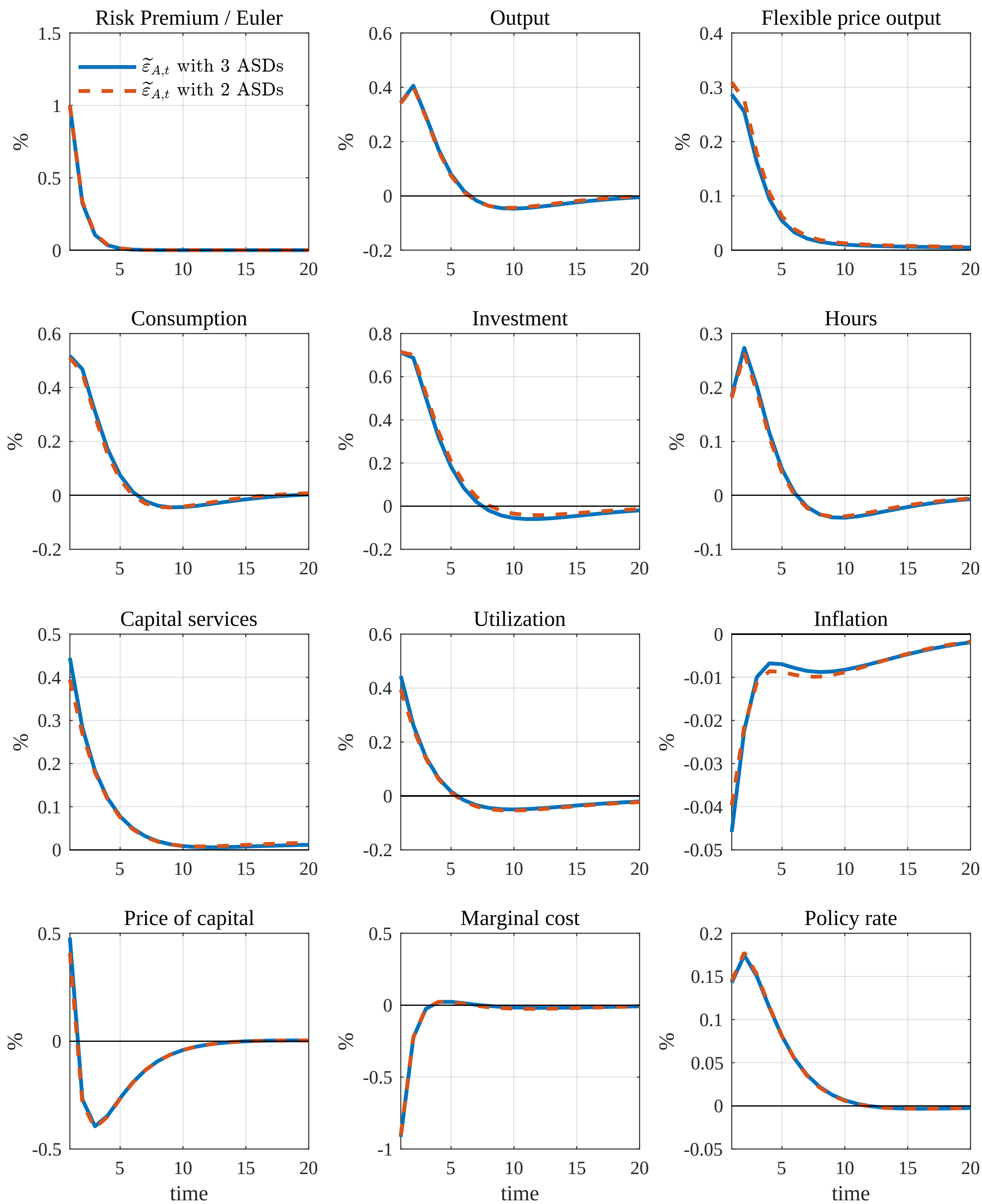
Notes. The table provides the contributions (in percent) of the different structural disturbances to the variance of the observable variables, across different model specifications. The ASD specifications are the ones chosen by our model selection procedure. y stands for log output; c for log consumption; i for log investment; l for hours; w for log wage rate; r^k for rental rate on capital; q for the log price of capital; z for the utilization rate; μ^p for the price mark-up; k^s for log capital used in production; and k for log installed capital. Structural disturbances are defined as follows. ε_a: TFP; ε_g: government expenditures; ε_r: monetary policy; ε_p:price mark-up; ε_w: wage mark-up; ε_b: risk premium; ε_i: investment; $\widetilde{\varepsilon}_A$: agnostic Euler; $\widetilde{\varepsilon}_B$: agnostic investment-modernization; and $\widetilde{\varepsilon}_C$: capital-efficiency wage mark-up.

Specifications with and without restrictions on ASDs. Table E.14 also compares structural parameter estimates of concise ASD models chosen by our model selection procedures with those that still allow ASDs to enter all equations. The parameter estimates are fairly similar. IRFs for the included regular structural disturbances are also quite similar.

That is not always the case for the IRFs of the agnostic disturbances themselves. The IRFs for some variables do differ between the concise and the fully unrestricted ASD specification. Given the misspecification results of Appendix D, it is not surprising that different empirical specifications lead to different results. Another issue with the fully unrestricted ASD specification is that it estimates a large number of coefficients which complicates generating an accurate posterior with Monte Carlo Markov Chain algorithms. Especially, for the 3-ASD fully unrestricted specification, the Brooks-Gelman statistics did not look particularly good for some of the coefficients associated with the agnostic disturbances. Thus, we prefer the concise ASD specifications.

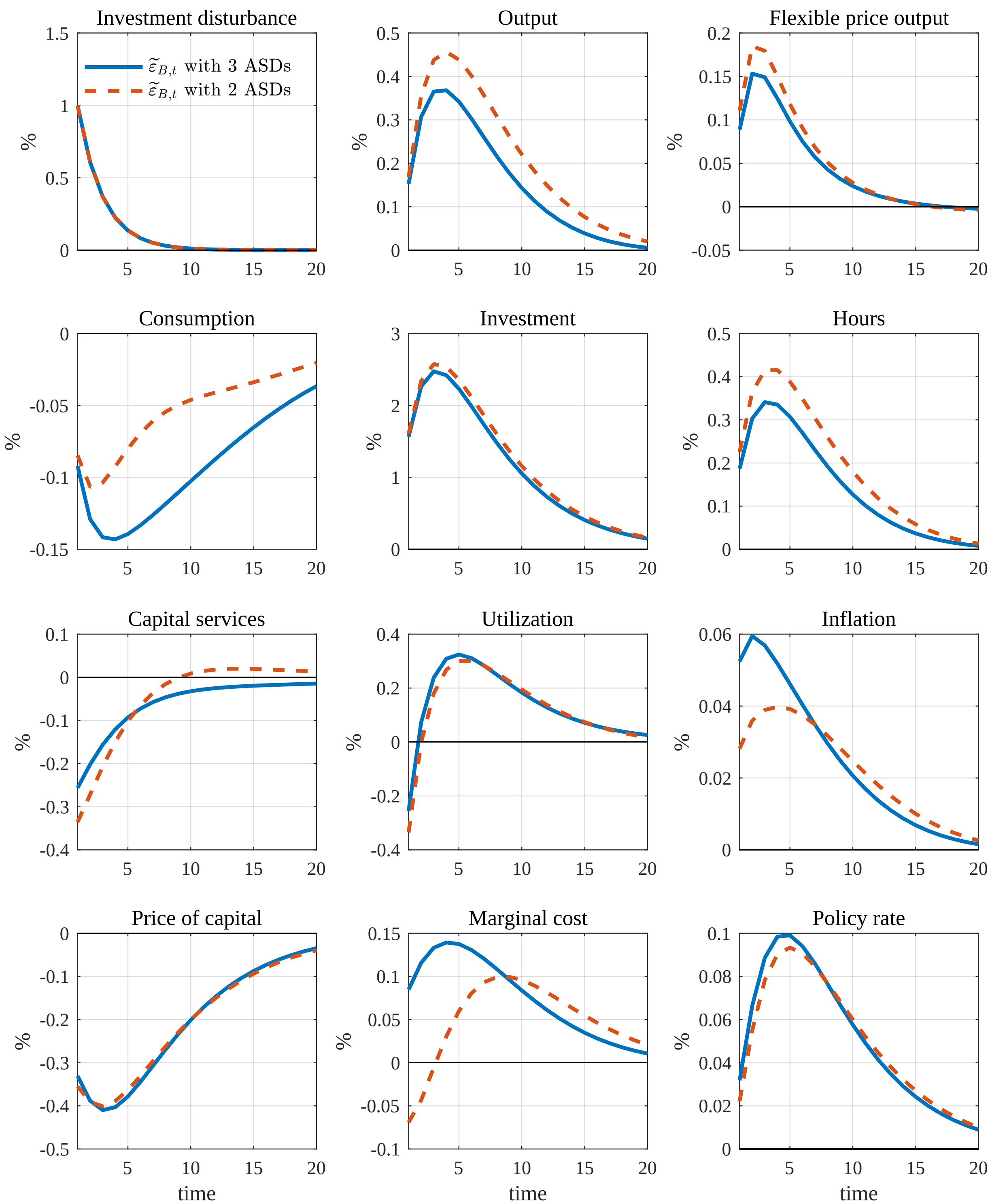
Specifications with two and three ASDs. Tables E.15 and E.16 provide the role of the regular and agnostic disturbances for the fluctuations of a wide range of variables. In addition to the results of the SW specification, it also shows the results for the two-ASD and three-ASD specification chosen by our specific-to-general model selection procedure. It shows that the results are very similar for the two chosen ASD specifications. The same conclusion can be drawn from Figures E.9 and E.10 that plot the IRFs for two agnostic disturbances.

Figure E.9: IRFs of the agnostic Euler disturbance: 2 versus 3 ASDs



Notes. These panels plot the IRFs of the agnostic disturbance $\widetilde{\varepsilon}_{A,t}$ that we interpret as a general Euler disturbance for the empirical specifications with two and three ASDs. Both specifications are chosen with the specific-to-general model selection procedure.

Figure E.10: IRFs of the agnostic investment-modernization disturbance: 2 versus 3 ASDs



Notes. These panels plot the IRFs of the agnostic disturbance $\widetilde{\varepsilon}_{B,t}$ that we interpret as an investment-modernization disturbance for the empirical specifications with two and three ASDs. Both specifications are chosen with the specific-to-general model selection procedure.

Additional results for $\widetilde{\varepsilon}_{A,t}$. Figure E.11 plots the IRFs associated with an innovation in the agnostic Euler disturbance for our 3-ASD benchmark specification and also when the coefficient of this agnostic disturbance in the capital valuation equation is equal to zero. A preference disturbance does not show up in this equation and a bond risk-premium disturbance does.[71] The IRFs are very similar, which confirms our claim that the coefficient in the capital valuation equation is quantitatively not very important. This does not mean that the ASD is a preference disturbance, since the ASD shows up in the investment equation whereas a preference disturbance does not.

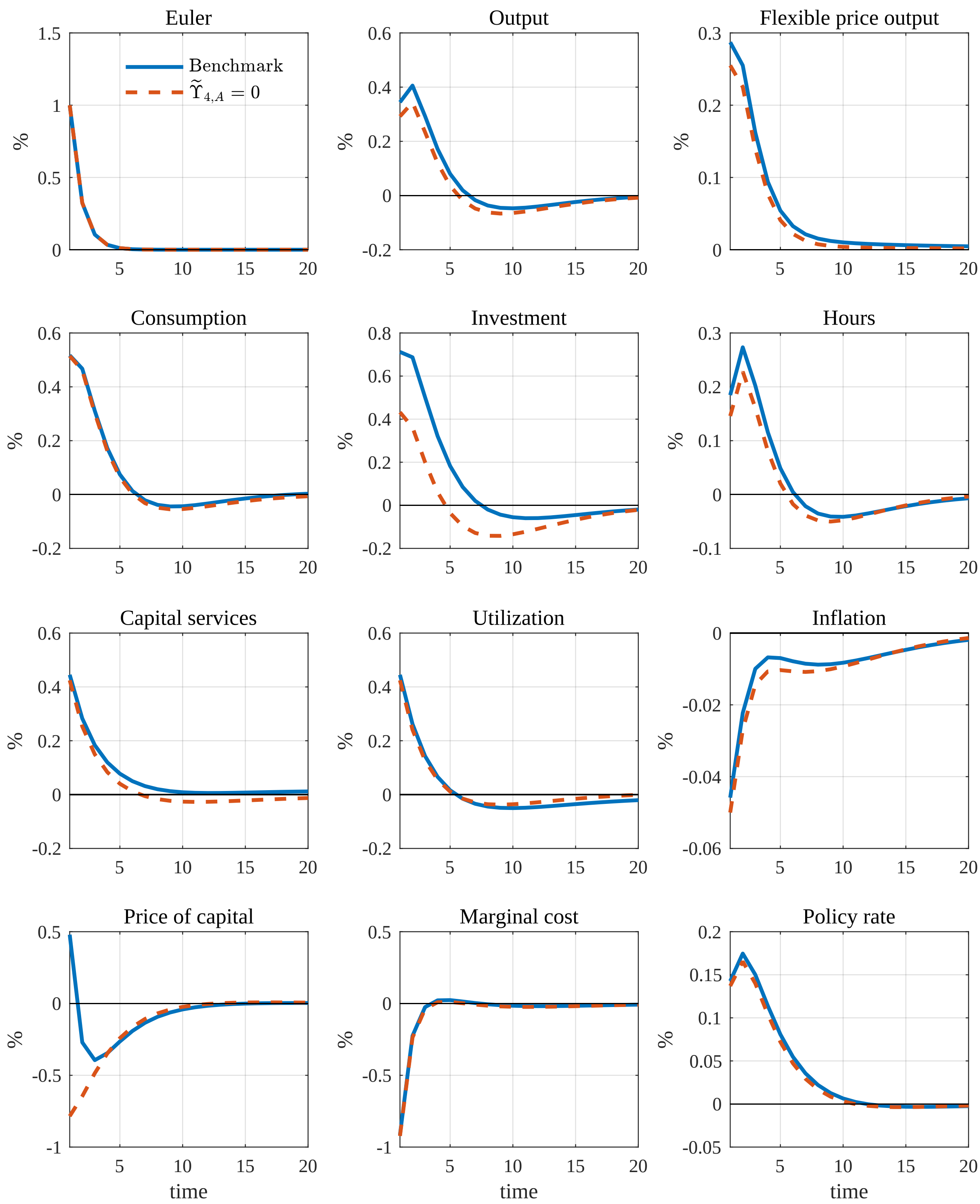
Figure E.12 plots the same IRFs when the coefficient of the agnostic Euler disturbance in the Taylor rule is set equal to zero. The figure shows that the direct response of the policy rate to a positive shock to this disturbance dampens the expansion and prevents an upsurge of inflation.

Figure E.13 plots the same IRFs when we set equal to zero the coefficients of the disturbance in the four equations that we ignored in the discussion of the agnostic Euler disturbance, namely, the overall budget constraint, the utilization, the price mark-up equation, and the rental rate of capital equation. The figure documents that the role of the agnostic disturbance through these equations is minor since the IRFs are overall quite similar to those of our benchmark specification.

Additional results for $\widetilde{\varepsilon}_{C,t}$. Figure E.14 plots the IRFs for our agnostic capital-efficiency wage mark-up disturbance when the coefficient of this disturbance in the overall budget constraint is set equal to zero. The figure documents that this has a minor impact on IRFs.

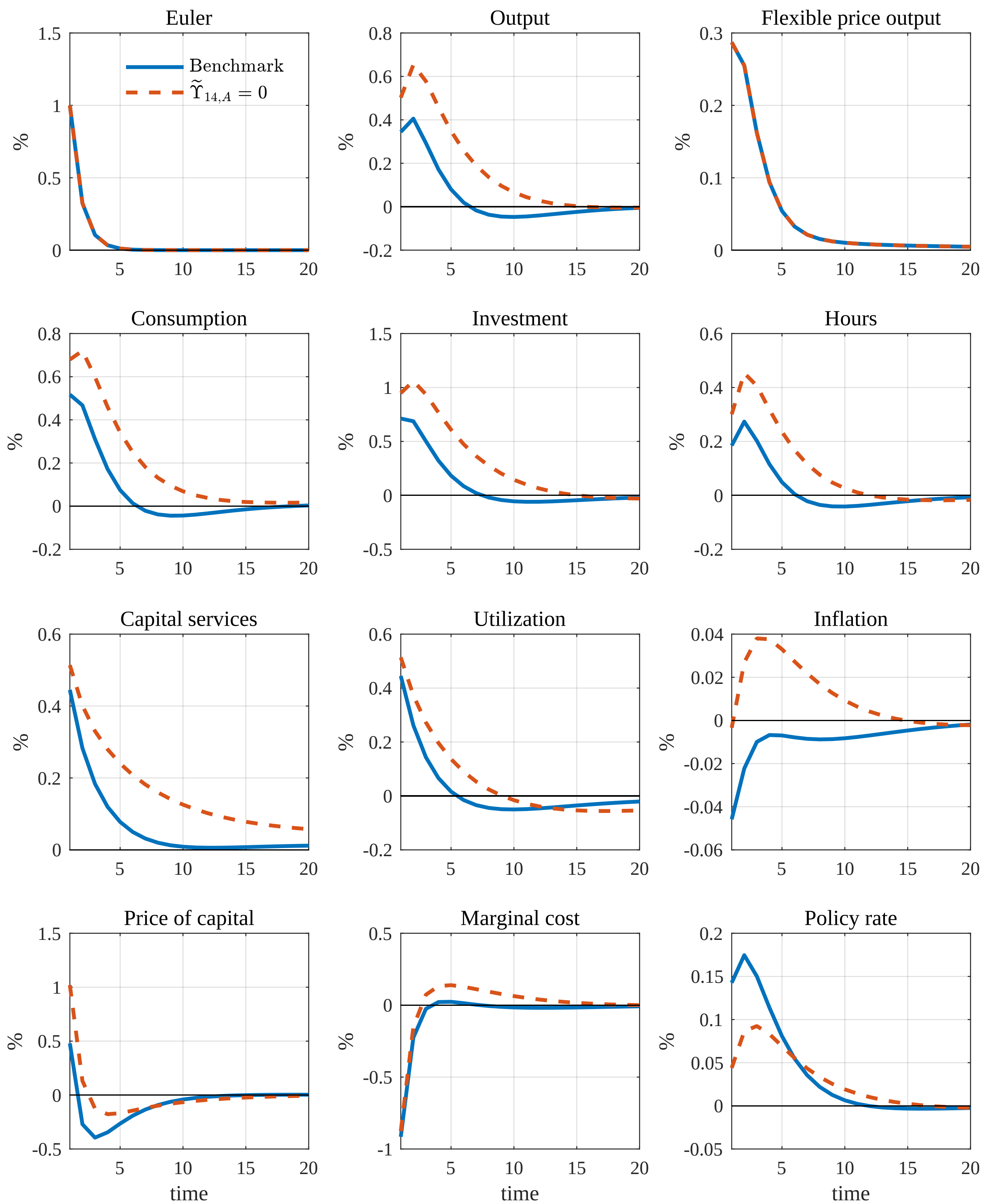
[71] Recall that the MRS has been substituted out of the capital valuation equation using the MRS of the bond Euler equation.

Figure E.11: IRFs of the agnostic Euler disturbance with restrictions I



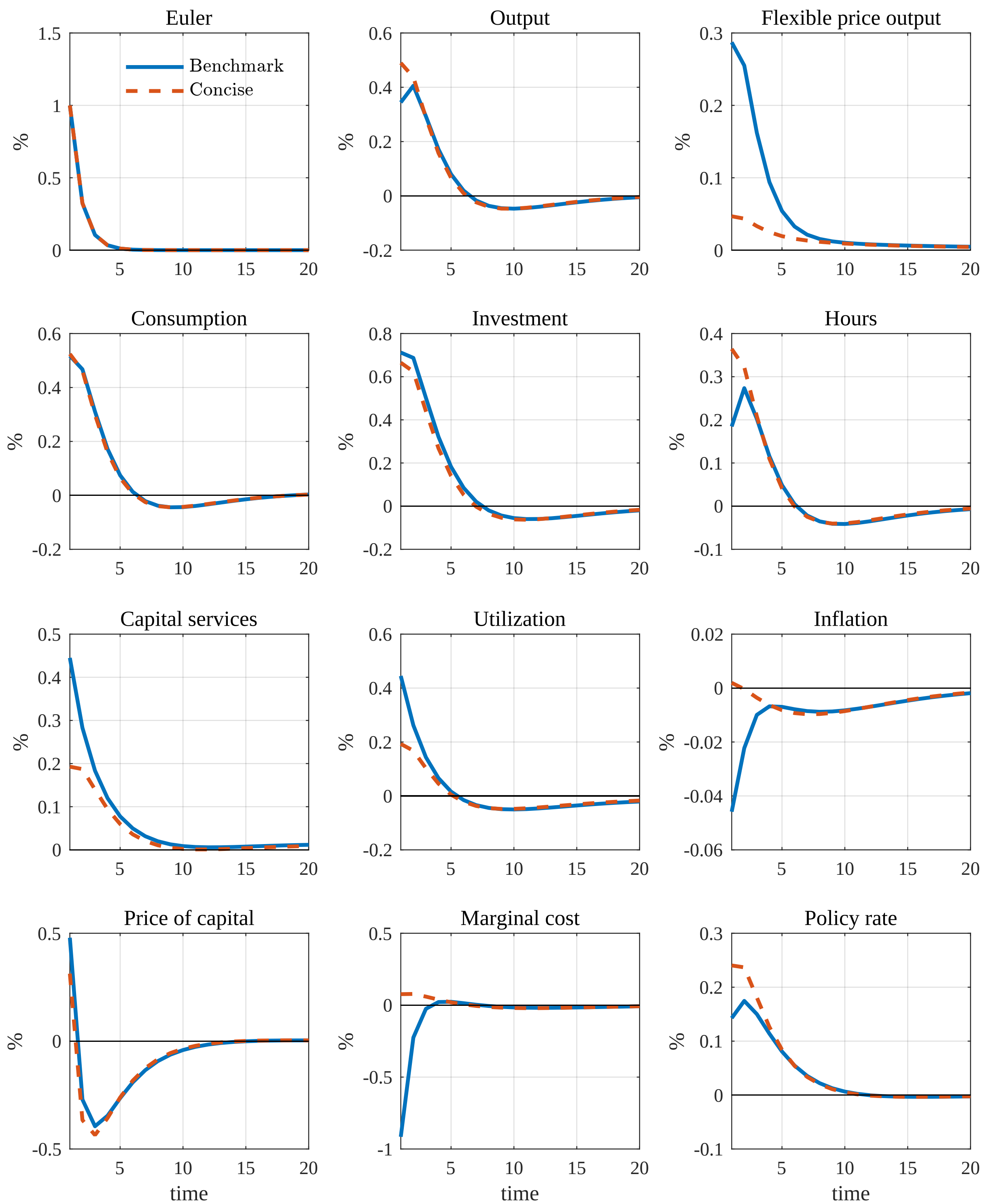
Notes. These panels plot the IRFs of the agnostic Euler disturbance for our benchmark specification and when the impact of this IRF through the capital valuation equation is set equal to zero.

Figure E.12: IRFs of the agnostic Euler disturbance with restrictions II



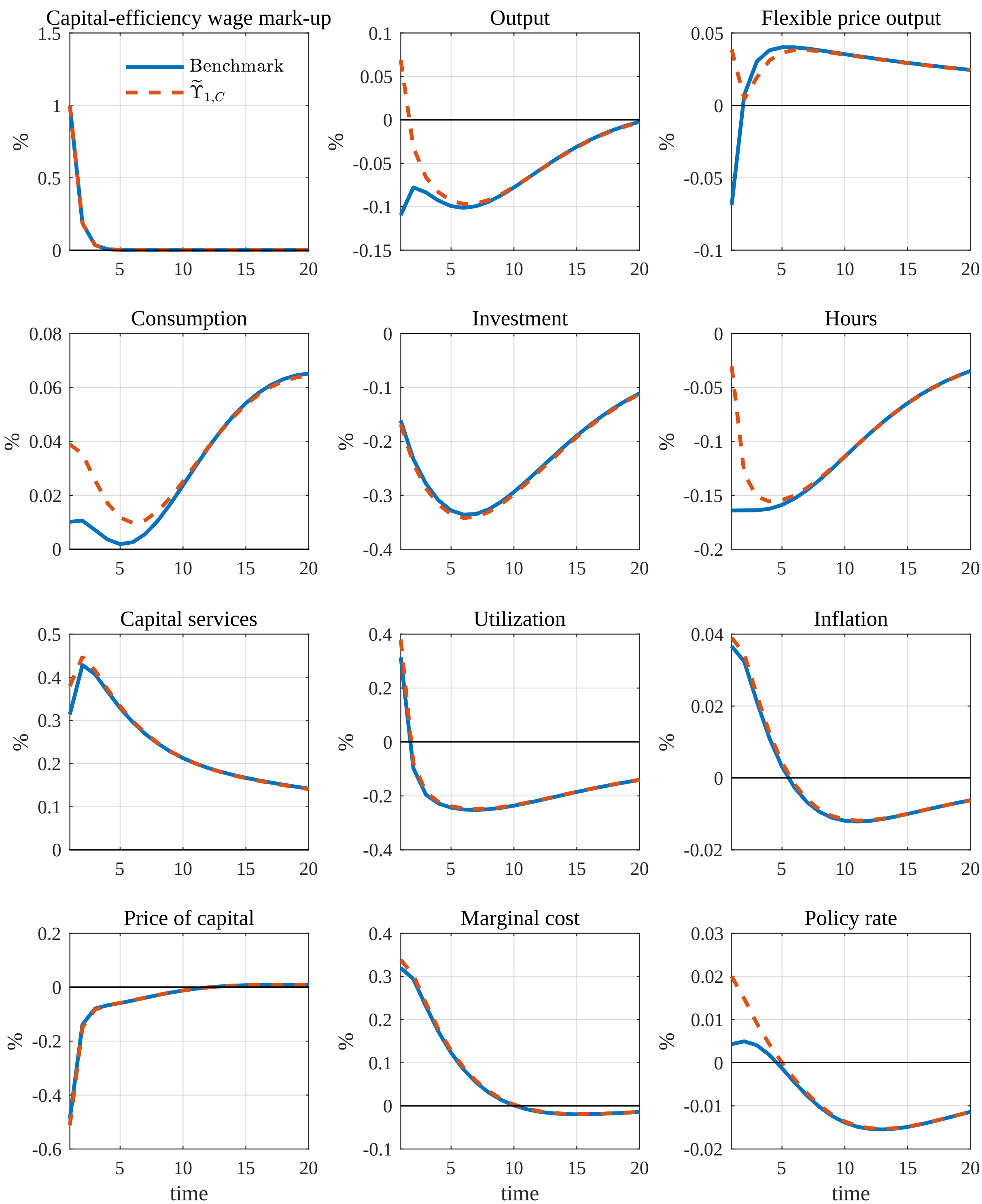
Notes. These panels plot the IRFs of the agnostic Euler disturbance for our benchmark specification and when the impact of this IRF through the Taylor rule is set equal to zero.

Figure E.13: IRFs of the agnostic Euler disturbance with restrictions III



Notes. These panels plot the IRFs of the agnostic Euler disturbance for our benchmark specification and when the impact of this IRF through the overall budget constraint, the utilization, the price mark-up equation, and the rental rate of capital equation is set equal to zero.

Figure E.14: IRFs of the agnostic capital-efficiency wage mark-up disturbance with restrictions



Notes. These panels plot the IRFs of the agnostic capital-efficiency wage mark-up disturbance for our benchmark specification and when the impact of this IRF through the overall budget constraint is set equal to zero.