Volatile Hiring: Uncertainty in Search and Matching Models*

Wouter Den Haan† Lukas B. Freund‡ Pontus Rendahl††

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Abstract
In search-and-matching models, the nonlinear nature of search frictions increases average unemployment rates during periods with higher volatility. These frictions are not, however, by themselves sufficient to raise unemployment following an increase in *perceived* uncertainty; though they may do so in conjunction with the common assumption of wages being determined by Nash bargaining. Importantly, option-value considerations play no role in the standard model with free entry. In contrast, when the mass of entrepreneurs is finite and there is heterogeneity in firm-specific productivity, a rise in perceived uncertainty robustly increases the option value of waiting and reduces job creation.

Keywords: uncertainty, search frictions, unemployment, option value.
JEL Classification: E24, E32, J64.

1. Introduction

There is a large empirical literature which demonstrates that economic volatility is time-varying and that heightened uncertainty negatively affects labor markets and macroeconomic activity, even when the rise in uncertainty is merely perceived.[1] Thus, increased uncertainty has been identified as one of the key contributors to historically significant increases in cyclical unemployment such as those occurring in during the COVID-19 pandemic (Baker et al., 2020; Leduc and Liu, 2020a) and the Global Financial Crisis (Baker et al., 2012).[2] Yet, several theoretical models and mechanisms predict the opposite. Precautionary motives call forth a rise in savings, which would be associated with increased investment. Also, limited liability means that firm owners' payoff function is convex, which implies that uncertainty increases firm equity value and makes investment more attractive. The famous "option-value-of-waiting" mechanism, however, does predict a negative relationship between elevated uncertainty and economic activity because higher (anticipated) uncertainty makes it more attractive to wait and postpone investment (cf. Bernanke (1983)).

The aim of this paper is to clarify the transmission mechanisms of uncertainty, and specifically the role of option-value considerations, in the canonical search-and-matching (SaM) model of the labor market. We build on Leduc and Liu (2016), who, in an important contribution, demonstrate that in a standard SaM model an increase in observed perceived uncertainty leads to an increase in the unemployment rate. In fact, they show that such a model can qualitatively match the empirical movements even under flexible prices and with prudent agents, a combination that,

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†London School of Economics, CEPR and CFM. Email: w.denhaan@lse.ac.uk. Web: wouterdenhaan.com. Postal address: London School of Economics and Political and Political Science, Department of Economics, Houghton Street, London WC2A 2AE, UK.

‡University of Cambridge. Email: lukas.beat.freund@gmail.com. Web: lukasfreund.com. Postal address: Faculty of Economics, Sidgwick Avenue, Cambridge CB3 9DD, United Kingdom. Corresponding author.

††Copenhagen Business School, CEPR and CFM. Email: pontus.rendahl@gmail.com. Web: sites.google.com/site/pontusrendahl. Postal address: Copenhagen Business Schoool, Department of Economics, Porcelaenshaven 16A, 2000 Frederiksberg, Denmark.

[1]For excellent surveys of the literature, see Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020). Also see, however, Berger et al. (2020) for an argument that it is realized volatility, rather than the perception of a more uncertain future, that matters for adverse economic consequences.

[2]Key references on the empirical effects of uncertainty include Jurado et al. (2015) and Baker et al. (2016). Regarding the effects of uncertainty on (un-)employment specifically, see Caggiano et al. (2014) and Leduc and Liu (2016). Freund and Rendahl (2020, Appendix A) document that elevated uncertainty causes not only the unemployment rate but also vacancy to respond in a manner that is consistent with the predictions of the search-and-matching frameworks explored in this paper.

by itself, typically pushes economic activity in the opposite direction.[3] Leduc and Liu (2016) do not bring to the surface what mechanism lies behind this result. But they conjecture that search frictions in the labor market produce an option-value channel that helps rationalize the adverse effects on economic activity of elevated uncertainty.

The first contribution of this paper is to provide an in-depth analysis of the effects of increased volatility in SaM models, examining in particular whether there is an option-value channel. Since job creation is very much like an irreversible investment, as the associated costs are not refundable, the option-value channel is a sensible candidate to consider. Nonetheless, the nonlinear aspects of the standard SaM model themselves do *not* lead to any option value of waiting. The reason is that the free-entry assumption implies that the expected value of vacancy posting is, and will always remain, equal to zero. Hence, there is no point in waiting.

We show that Leduc and Liu's (2016) result, that an increase in perceived uncertainty leads to an increase in the unemployment rate in a standard SaM model with flexible prices, depends crucially on the assumption that wages are determined by Nash bargaining, an assumption that is often adopted in the SaM literature. However, many other types of wage setting assumptions are possible. Specifically, we show that changes in perceived uncertainty have *no* effect on job creation when wages are linear in productivity. Nonlinear matching friction do imply that the *average* value of labor market tightness – the number of vacancies relative to the number of unemployed workers searching for a job – is elevated during periods of higher *realized* volatility. Under Nash bargaining, this improves workers' bargaining position and raises the average wage. Even if higher volatility is not realized, the increase in wages is driven by what agents *expect* to happen when perceived uncertainty increases. As a result, the firm value falls, which reduces job creation.

The literature often focuses on the impact of an increase in *perceived* uncertainty; that is, the impact that is solely due to beliefs, not to an actual increase in volatility. Our second contribution is to highlight the importance of analyzing the impact of *realized* increases in volatility (measured as the impact averaged over all possible realizations). As mentioned above, how perceived uncertainty affects the economy depends on what agents *expect* to happen during the period of heightened volatility. Furthermore, both in the standard SaM framework and in the modifications we consider, we find that *even if* the anticipation of uncertainty itself does have a non-zero impact on the economy, these effects are small compared to those induced by realized volatility. In addition, the effects of realized and perceived volatility can differ in sign along the IRF over at least some horizons.

Our third contribution is to demonstrate how wait-and-see considerations can be introduced into the SaM framework. Our starting point is to observe that virtually all SaM models assume that there are always enough potential entrepreneurs available to drive the expected profits of job creation to zero. We first highlight that an option-value channel is in principle possible by relaxing the free-entry condition and assuming that the mass of entrepreneurs is finite.[4] However, the resulting channel is only operative under restrictive assumptions. In particular, the free-entry condition must be binding in some states, such that firms make zero profits, but not in others, such that firms make positive profits. In a second step, we therefore add heterogeneity in idiosyncratic firm-productivity alongside the assumption of a finite mass of entrepreneurs. In the resulting SaM framework, there is a time-varying measure of entrepreneurs, namely those with a sufficiently high productivity draw, that expect to make strictly positive profits when they post a vacancy. With this relatively simple modification, the model robustly predicts that perceived uncertainty leads to a postponement of job creation. The reason is that an expected increase in future volatility increases an unmatched entrepreneur's (future) chance of having a productivity draw for which expected profits of vacancy-posting are strictly positive, whereas the downside risk is not affected since unmatched entrepreneurs can always choose to stay out of the market. Our proposed model remains tractable and can be solved by (higher-order) perturbation methods.

Our study is connected to two broad strands of the economic literature, namely, analyses considering frictional labor market models and the effects of uncertainty, respectively. While each of these literatures is vast in scope, we briefly comment on the most closely related studies. In particular, Bloom (2009) offers a seminal examination of the

[3] The decline in demand for consumption goods induced by elevated uncertainty leads to reduced economic activity when prices are sticky (cf. Basu and Bundick (2017)). In fact, an additional contribution of Leduc and Liu (2016) is to show the importance of embedding their SaM model into a New Keynesian framework with price rigidities. Together with nonlinear household utility, these give rise to an aggregate demand channel which ensures that the model can generate quantitatively substantial effects following uncertainty shocks. To make our analysis more transparent, we abstract from price stickiness and associated demand effects.

[4] We share with Coles and Kelishomi (2018) and Leduc and Liu (2020b) an emphasis on the restrictive nature of the standard free-entry condition in the SaM model. We differ from these papers in that we concentrate on the implications of free entry for the effects of uncertainty shocks and, in particular, the presence or absence of option-value effects.

sort of real-options effects analyzed also in this paper. His model incorporates option-value considerations in hiring and (physical) investment space due to non-convex adjustment costs. We consider a particularly prominent variant of adjustment costs, namely search frictions in the labor market, and identify the conditions under which an option-value effect materializes, and when it does not.

Considering studies that share this focus beyond Leduc and Liu (2016), the most closely related paper is Schaal (2017). That paper develops a model with multi-worker firms that are heterogeneous in productivity and which are subject to an endogenous linear hiring cost at the firm level.[5] In similarity to our proposed model, the resulting irreversibility gives rise to an option value of waiting. In contrast to our approach, the free-entry condition binds in every state of the world. The reason that the value of vacancy posting nonetheless varies over time in Schaal's (2017) setup is that firms operate a decreasing returns to scale technology, and the free-entry condition obtains at the level of the (multi-worker) firm rather than at the vacancy. We, by contrast, stay as close as possible to the canonical Diamond-Mortensen-Pissarides assumptions of constant returns to scale in production and random search, while restoring an option-value channel.[6]

The next section lays out a standard SaM model and its calibration. Section 3.1 reports the effects on the economy both to an increase in perceived and realized uncertainty, respectively. Sections 3.2-3.4 analyze these results in detail. Section 4 discusses our modifed SaM model in which there is an option value of postponing job creation. The last section concludes.

2. Theoretical Framework

We begin by summarizing a basic search-and-matching (SaM) framework. This is the same model as studied by Leduc and Liu (2016), except that we restrict ourselves to the flexible-price version and assume the representative household to be risk neutral. Both assumptions are common in the matching literature. For us they have the benefit of making the analysis more transparent. Specifically, as shown in Bernanke (1983) and our example in section 3, the option value of waiting does not rely on risk aversion.[7] Nor are sticky prices necessary.

Model equations.. The model is characterized by four equations in four unknown variables, J_t, w_t^N, θ_t, and n_t.

$$J_t = \bar{x}z_t - w_t^N + \beta E_t\left[J_{t+1}(1-\delta)\right], \tag{1}$$

$$w_t^N = \omega\left(\bar{x}z_t + \beta(1-\delta)\kappa E_t\left[\theta_{t+1}\right]\right) + (1-\omega)\chi, \tag{2}$$

$$\kappa = h(\theta_t)J_t, \tag{3}$$

$$n_t = (1-n_{t-1}+\delta n_{t-1})f(\theta_t) + (1-\delta)n_{t-1}. \tag{4}$$

Here, J_t denotes firm value; z_t the (exogenous) labor productivity; w_t^N the wage rate based on Nash bargaining; θ_t labor market tightness; and n_t the mass of productive relationships (or employment).[8]

The firm value, J_t, is simply the present-discounted value of firm profits, using the household's discount factor, β, and taking into account that firms separate at the exogenous rate δ. The parameter χ in the wage expression represents the benefit that accrues to workers if they are not employed. If the bargaining weight of the worker, ω, is equal to zero, wages are simply equal to χ, which therefore serves as a floor on wages. If the bargaining weight of the worker, ω, exceeds zero, however, the wage rate increases with the firm's marginal revenues, $\bar{x}z_t$, as well as with the expected value of future market tightness.[9]

[5] For a related approach, see Riegler (2019).

[6] While in the main text we interpret our model as describing one-worker firms, in online appendix OE we show that there exists an equivalent representative multi-worker firm representation – just as it does for the standard SaM model.

[7] Freund and Rendahl (2020) explain how risk aversion can lead to a larger impact of uncertainty shocks through changes in the required risk premium. We further discuss the role of risk aversion in online appendix OB.

[8] In the model of Leduc and Liu (2016), final good producers earn a markup, which means that the relative price of intermediate goods, $\bar{x}$, is less than one. In this version with flexible prices, the level of the markup is not relevant, but we retain the parameter $\bar{x}$, so that our calibration is as close as possible to Leduc and Liu's (2016).

[9] See online appendix OA.1 for a derivation.

The job finding rate, $f(\theta_t)$, and the hiring rate, $h(\theta_t)$, are determined in the matching market. The number of matches in period t is determined by the matching function $m_t = \psi v_t^{1-\alpha}(u_t^s)^{\alpha}$, where v_t denotes the number of vacancies and u_t^s the measure of workers searching for a job; that is, $u_t^s = 1 - n_{t-1} + \delta n_{t-1}$. This functional form implies that $h_t = \frac{m_t}{v_t} = \psi\theta_t^{-\alpha}$ and $f_t = \frac{m_t}{u_t^s} = \psi\theta_t^{1-\alpha}$, where θ_t indicates labor market tightness, $\theta_t = \frac{v_t}{u_t^s}$.

The cost of posting a vacancy is equal to κ and its expected benefit is equal to the product of the hiring rate, h_t, and the value that accrues to a successful entrepreneur, J_t. The standard assumption of SaM models is that there is a potentially infinite number of entrepreneurs. This means that κ has to equal $h_t J_t$ as indicated by the free-entry condition in equation (3). The free-entry condition implies that an increase in J_t leads to a lower value of h_t, which is accomplished by an inflow of additional entrepreneurs into the matching market, i.e., an increase in v_t and a reduction in θ_t.

Finally, the value of the exogenous variable z_t is determined by the following process

$$z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \sigma_{t-1}\varepsilon_{z,t}, \tag{5}$$

$$\ln(\sigma_t) = (1-\rho_\sigma)\ln(\sigma) + \rho_\sigma \ln(\sigma_{t-1}) + \sigma_\sigma \varepsilon_{\sigma,t}, \tag{6}$$

where $\varepsilon_{z,t}$ and $\varepsilon_{\sigma,t}$ are *iid* standard Normal processes. The steady-state value of productivity, $\bar{z}$, is normalized to unity. Uncertainty shocks are associated with changes in $\varepsilon_{\sigma,t}$. This specification of the stochastic processes is common in the literature but deviates from Leduc and Liu (2016) in two respects. First, the process for z_t is in levels rather than in logarithms to prevent the expected value of productivity to differ from its steady-state through a Jensen's inequality effect. Second, we use the timing assumption common in the uncertainty literature according to which volatility shocks have a delayed impact on the distribution of productivity shocks (e.g., Bloom (2009)). We do so to underscore that real options effects are absent even under a timing assumption that is, in principle, favorable to wait-and-see effects (cf. Schaal (2017, footnote 12)). As in Leduc and Liu (2016), we specify the process for σ_t in logs to ensure that the standard deviation remains positive.

Calibration and solution method.. The calibration follows Leduc and Liu (2016) as closely as possible. The calibrated parameter values and the associated targets/outcomes are reported in Table 1. We also use the same solution method, that is, third-order pruned perturbation.

3. Volatility in the standard search-and-matching model

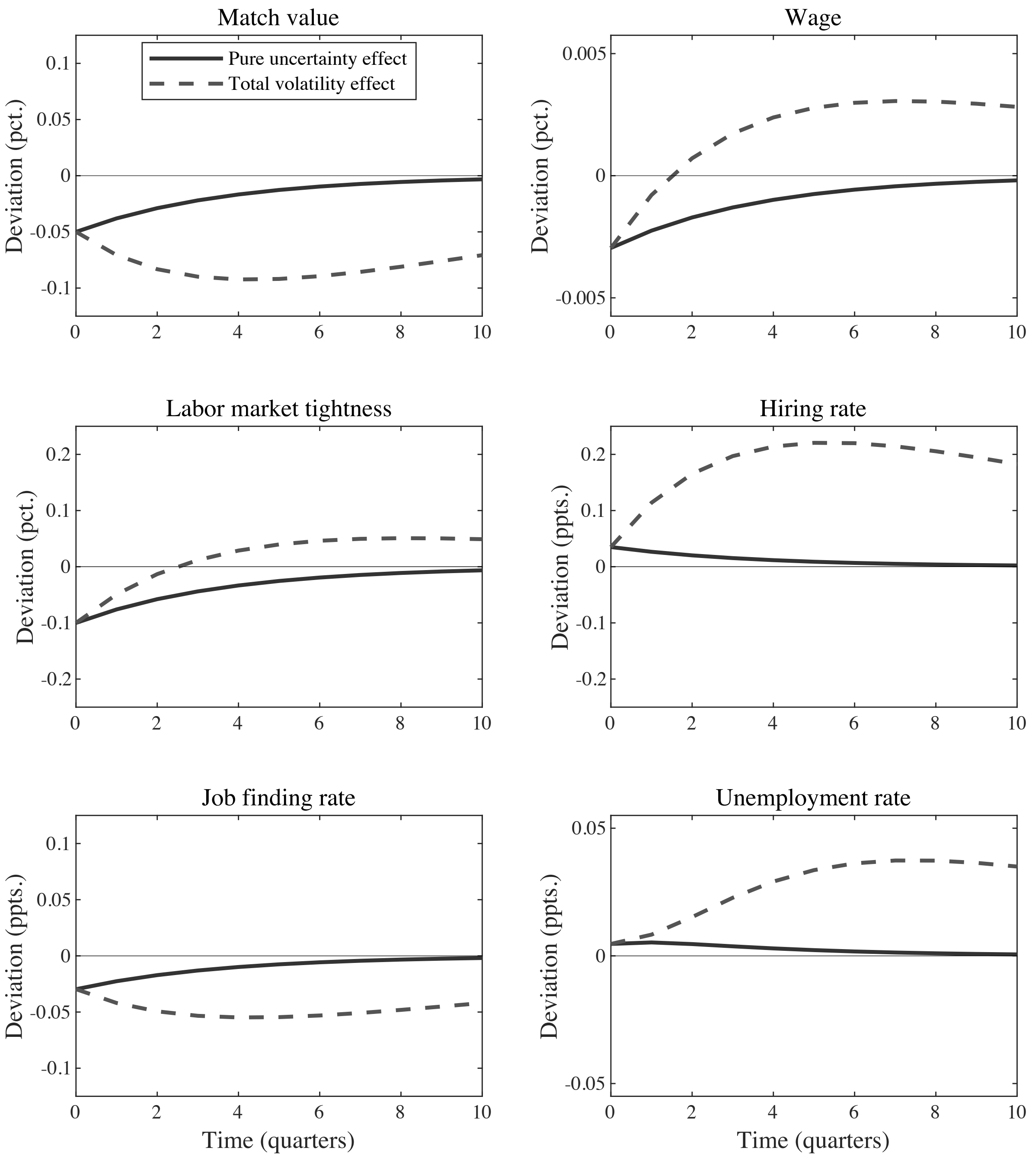
The main objective of this section is to present and analyze the effect of volatility shocks in the standard search-and-matching (SaM) model. We proceed in four steps. First, we illustrate IRFs of the baseline model, and outline a distinction between the total volatility effects and those that arise purely from anticipation. As we will see, increased uncertainty generally leads to a decline in firm value and a rise in unemployment. However, the results are more complex with respect to variables such as labor market tightness and wages. Next, we provide a simple two-period version of the model to illustrate when an option-value channel may emerge, and show that these conditions are not met in the SaM framework. Thus, the decline in economic activity revealed by the IRFs is not due to an option-value effect. Third, we show that *all* anticipation effects disappear once we replace Nash bargaining with a wage rule that is linear in productivity. Lastly, we explain why Nash bargaining can have non-trivial implications for the transmission of uncertainty shocks.

3.1. *Impulse response functions*

Figure 1 plots the IRFs for an uncertainty shock, that is, an increase in $\varepsilon_{\sigma,t}$. We plot two different types of IRFs. The first, the *total volatility* IRFs, of variable x_t is the standard IRF that plots $E_\tau[x_{\tau+j}]$, where τ is the period the shock occurs and $j = 0, 1, \cdots$. These IRFs describe what happens on average (or, equivalently, in *expectation*) during periods of heightened volatility. Whereas in linear models the impact of a period-τ shock on variables in subsequent periods does *not* depend on realizations of future shocks, in nonlinear models it does. This means that one has to integrate over all possible future realizations to calculate this *expected* impact of a period-τ shock.[10]

[10] The starting point does potentially matter in a nonlinear model. Here we follow the literature and suppose that the shock occurs after a long period during which no shocks have materialized at all (cf. Born and Pfeifer (2014)). The total volatility IRFs are calculated using the technique of

Figure 1: IRFs for uncertainty shock in standard SaM model under Nash Bargaining



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The "pure uncertainty" IRFs display how the economy responds when agents think volatility will increase, but the higher volatility actually never materializes.

Table 1: Calibrated parameters

Parameter	Interpretation	Source/target	Wage Nash	Wage Linear
β	Discount factor	Ann. interest rate of 4%	0.99	0.99
ψ	Efficiency of matching	Unemployment rate of 6.4%	0.645	0.645
$\bar{x}$	Markup	Markup of 11%	0.9	0.9
δ	Separation rate	JOLTS database	0.1	0.1
ω	Workers barg. power	Steady-state wage relation	0.5	0.915
α	Elasticity of matching	Petrongolo and Pissarides (2001)	0.5	0.5
κ	Vacancy posting cost	2 % of steady-state output	0.14	0.14
χ	Disutility of working	$\eta_{\theta,z}$ equal to 7.051	0.751	0.645
ρ_z	Persist. of agg. product.	Leduc and Liu (2016)	0.95	0.95
ρ_σ	Persist. of uncertainty	Leduc and Liu (2016)	0.76	0.76
σ_z	Std. agg. product. shock	Leduc and Liu (2016)	0.01	0.01
σ_σ	Std. uncertainty shock	Leduc and Liu (2016)	0.392	0.392

Notes. This table lists the parameter values of the baseline SaM model with both types of wage setting. One period in the model corresponds to one quarter. There are some slight unavoidable differences with the calibration procedure of Leduc and Liu (2016). For instance, with risk neutrality, the calibrated value of the disutility of labor parameter, χ, is slightly different than with log utility. Also, with utility linear in consumption there is no difference between disutility of labor and unemployment benefits and our χ parameter captures both.The targeted value of $\eta_{\theta,z}$, the steady-state elasticity of labor market tightness with respect to aggregate productivity, is implied by Leduc and Liu's (2016) calibration for the model with Nash bargaining and linear utility. Parameter values are rounded to three decimal places.

The second type of IRFs, the *pure uncertainty* IRFs, plots the response of the economy when agents perceive an increase in future volatility, but this increase never materializes. For an IRF that uses the stochastic steady state as the starting point, this means that agents think σ_t is higher than normal during the periods following the shock and act accordingly, but productivity, z_t, remains at its steady-state value. Thus, the pure uncertainty IRF measures the effects of an increase in *purely anticipated* uncertainty, as examined also by Leduc and Liu (2016). These effects arise solely due to agents' responses to changed expectations about the future as described by the total volatility IRFs. Thus, the latter type of IRFs is essential to understand the first kind.[11]

The key observations about figure 1 are as follows. First, the value of a firm falls and the unemployment rate increases. This is true for both types of IRFs. Second, there are important qualitative differences between the two types of IRFs. Whereas the pure uncertainty IRFs follow the usual monotone pattern, the total volatility IRFs display an inverted u-shape. Moreover, for the wage rate and tightness variable, the two types of IRFs even have different signs at some horizons.[12] Both types of IRFs take on negative values initially for these two variables, but the response of the total volatility IRFs turns positive soon after the shock occurs whereas this is not the case for the pure uncertainty IRFs. As explained in the next section, this observation is important to understand why the firm value drops in the matching model with Nash bargaining when volatility increases or is anticipated to do so.

3.2. *The evasive option value*

To understand the IRFs presented in the last section, we make use of a simple example to illustrate why and when an increase in uncertainty increases the option value of postponing workforce investment. As we will see, there cannot be an option-value channel operating within the standard SaM framework, and the example developed here is useful to make this clear.

Andreasen et al. (2018).

[11] This relationship is also important to understand first-order moment "news" shocks, that is, how the economy responds in period t to news that productivity will be higher in the future requires understanding how the economy is expected to respond to such an increase in productivity.

[12] By construction, the two IRFs take on the same value in the period when the shock occurs, since our timing assumption implies that uncertainty shocks have no effect in that period. Thus, the impact-period responses for both types of IRFs are purely based on expectations on what will happen and those expectations are the same.

Option value of postponing investment. The option value to wait is most transparent under risk neutrality, as risk aversion will add additional aspects to the analysis, such as precautionary savings and changes in risk premia. Hence, we consider a risk neutral agent. This agent can choose between the following two investment paths. The first possibility consists of investing immediately and earning a known return R_1 in the first period and a stochastic return R_2 in the second period. The latter return will only become known in period 2. Alternatively, the agent can postpone making a decision. In this case, she would instead bring the money to the bank in the first period and earn a return equal to $R^* < R_1$. In the second period, the agent will invest in the project only if $R_2 > R^* \geq 0$. The expected values of the two strategies – *commit* and *wait* – are given by

$$J_{\text{commit}} = R_1 + \beta E[R_2], \tag{7}$$

$$J_{\text{wait}} = R^* + \beta E[\max\{R_2, R^*\}]. \tag{8}$$

How does increased volatility, i.e., an increase in the variance of R_2, affect the entrepreneur's choice when we keep the *expected value* of R_2 the same? It does not affect the value of J_{commit}. However, it increases the value of J_{wait}. The reason is that by waiting the entrepreneur is ensured of a minimum return, R^*, but she benefits from the higher upward potential of the investment project.

We want to highlight two features that are important. First, the decision is *irreversible*. That is, if the entrepreneur starts the project in period 1, then she cannot unwind the project in period 2 and get a refund. Second, the projects are *mutually exclusive*. That is, the entrepreneur has to adopt either the commit or the wait strategy.

Option value of waiting in search-and-matching models.. For comparison purposes, consider a two-period version of the standard SaM model.[13] An entrepreneur who invests by creating a vacancy in period 1 faces the cash flow

$$-\kappa + h_1 \underbrace{(R_1 + \beta E[R_2])}_{J_1}, \tag{9}$$

where R_t is now equal to profits net of wages. An entrepreneur who waits has no income in period 1 and we get

$$0 + \beta E[\max\{-\kappa + h_2 \underbrace{R_2}_{J_2}, 0\}]. \tag{10}$$

Investments are irreversible in the SaM model, since κ is paid upfront. Does this mean that individual entrepreneurs in SaM models have a benefit of waiting when the expected volatility of period-2 profits increases keeping their expected value constant? The answer is no. The free-entry condition implies that expected profits are equal to zero in every time period and in every state of the world; that is, $-\kappa + h_t J_t = 0$, $t = 1,2$. Since profits from vacancy-posting are expected to always be equal to zero, the upward potential that increased the value of waiting in the example discussed above does not exist here. That is, with free entry the last two equation can be written as

$$-\kappa + h_1 J_1 = 0, \tag{11}$$

$$\beta E[\max\{-\kappa + h_2 J_2, 0\}] = \beta E[\max\{0, 0\}] = 0. \tag{12}$$

Thus, although job creation is irreversible, it is not sufficient to generate an option-value channel.[14]

[13] What matters for economic activity in a standard SaM model with exogenous separation are the expected profits from posting a vacancy, not those of an existing match. Thus, we consider the decision of an unmatched entrepreneur.

[14] Irreversibility refers to the posting costs. Here, we have assumed that the entrepreneur who invests in period 1 will have some positive cash flow in period 2 for sure ($R_2 \geq 0$). That is, there is no incentive to end the relationship early endogenously. But R_2 could be negative, for example, with sticky wages. Allowing for endogenous discontinuation means that the net present value of the surplus accruing to the entrepreneur investing in period 1 is given by

$$-\kappa + h_1 R_1 + \beta E[\max\{R_2, 0\}] = -\kappa + h_1 J_1 = 0.$$

The convexity introduced by endogenous job destruction implies that an increase in anticipated uncertainty would *raise* the value of J_1, which in turn would lead to an increase in vacancies in period 1. That is, the outcome is the opposite of that predicted by an option value of waiting mechanism.

Key for the result that the expected value of vacancy posting is zero is that investing now and waiting are *not* mutually exclusive. That is, posting a vacancy this period does not prevent vacancies from being posted next period. It would not make a difference if these choices were mutually exclusive for the entrepreneur herself, that is, if one assumed that each entrepreneur can be involved in one project only. The reason is that there are always *other* entrepreneurs who can pursue the alternative choices, exhausting all positive profits. Thus, mutual exclusivity applies – respectively, does not apply – to the economy *as a whole*, and not to individual agents.

In section 4, we will show that it is possible for the SaM model to have an option-value mechanism if one assumes that each matched entrepreneur cannot be involved in more than one project *and* the mass of entrepreneurs is finite. This is sufficient to create an environment in which projects are mutually exclusive and expected profits are potentially positive.

Figure 1 demonstrates that there is one aspect of the properties of the SaM model developed in section 2 that is quite different from the analysis based on the simple two-period setup. Specifically, figure 1 documents that the value of a match, J_t, declines in response to an anticipated uncertainty shock, whereas the value of investing early in the two-period model, J_1, remains unaffected.

One might conjecture that the reason behind this decline in J_t is an increase in the option value of waiting (Leduc and Liu, 2016, p. 21). But J_t in the matching model corresponds to J_{commit} in the simple model; that is, to the value of investing *now*. In contrast, the idea of the option value to wait is that the value of waiting and potentially investing later increases. In the terminology of our stylized setup, an increase in uncertainty leads to an increase in J_{wait}, not to a decrease in J_{commit}.

3.3. *If it is not an option value, what is it?*

Our discussion above made clear that the environment of the standard aM model does not satisfy the conditions that generate an option value of postponing job creation. But it is still the case that volatility shocks lower the match value and increase the unemployment rate. The question is why does this happen, and can the reason still be given some option-value interpretation?

Note that in the two-period model, we assumed that $E[R_2]$, i.e., expected profits, remain the same when we increased the expected volatility. The same is true for expected values of future productivity in the full dynamic models. Thus, it must be the case that the behavior of wages is essential for understanding the results in figure 1.

The Nash bargaining assumption adopted in Leduc and Liu (2016) is just one of many possibilities and it is not an essential characteristic of the matching mechanism. Those essentials are, firstly, that neither workers nor entrepreneurs find a match with probability one. And secondly, that both sides face congestion effects, so that the probability of finding a match decreases if more of your ypre are searching; the matching function is concave in both arguments. To better understand the role of uncertainty in SaM models, we first consider the case in which not only the expected value of productivity but also the expected value of profits remains unchanged when volatility increases. This can be easily accomplished if one assumes that wages are a linear function of current productivity, z_t, only.[15] Specifically,

$$w_t = \omega \bar{x} z_t + (1-\omega)\chi. \tag{13}$$

Matching frictions and anticipated volatility changes. Under this wage rule one can derive a useful, analytical expression for J_t.

Proposition 1. *Suppose that wages are set by the linear wage rule given in equation (13), then*

$$J_t = \frac{(1-\omega)\bar{x}}{1-\beta(1-\delta)\rho_z} z_t - \frac{(1-\omega)\chi}{1-\beta(1-\delta)} + \frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{(1-\beta(1-\delta))(1-\beta(1-\delta)\rho_z)}. \tag{14}$$

Proof. See online appendix OA. □

[15] This linear specification can be motivated by an alternating-offers game (Hall and Milgrom, 2008; Freund and Rendahl, 2020). A key aspect of this game is that separation is not a credible threat. Consequently, agreement is reached within the period and market tightness does not affect the outcome. As long as agreement has not been reached, the worker is not working. The parameter χ captures the utility of not working during the negotiations. This wage coincides exactly with that of Jung and Kuester (2011), in which the Nash product, $(w_t-\chi)^{\omega}(\bar{x}z_t-w_t)^{1-\omega}$, is maximized.

Thus, J_t is a linear function of z_t. The formula immediately makes clear that an increase in *anticipated* volatility has no effect on J_t. If the anticipated increase in volatility does not materialize, then J_t will not change in subsequent periods either even when agents continue to anticipate higher uncertainty in the future. Consequently, none of the other variables will be affected either, as is documented in figure 2 which plots the two types of IRFs for an increase in uncertainty under the linear wage rule.[16]

The fact that the IRFs associated with an anticipated increase in volatility are zero in every period allows us to draw a strong conclusion. That is, the nonlinearity of the matching function by itself does *not* generate an employment effect in response to an increase in anticipated uncertainty. Consequently, there is also no option-value channel associated with the pure anticipation effect of an increase in uncertainty.

Still, increased volatility in productivity will make J_t more volatile, which in turn renders matching probabilities more volatile as well. We therefore explore next whether the nonlinearities of the matching function could be such that increases in volatility affect the expected values of employment during the period of elevated volatility.

Matching frictions and realized volatility changes. How can we expect an increase in the standard deviation of productivity shocks to affect values of key variables in the model during the period of higher volatility? Given the linearity of J_t, the total volatility IRF of J_t will also be zero. However, as demonstrated by figure 2, there are increases in the expected values of market tightness, θ_t, the hiring rate, h_t, and the unemployment rate, u_t. But the expected values of the job finding rate, f_t, are unaffected. To understand these results recall the expressions for θ_t, h_t, and f_t.

$$h_t = \frac{\kappa}{J_t}, \quad \theta_t = \left(\frac{\psi}{\kappa} J_t\right)^{\frac{1}{1-\alpha}}, \text{and} \quad f_t = \psi \left(\frac{\psi}{\kappa} J_t\right)^{\frac{1-\alpha}{\alpha}},$$

where α is the curvature parameter in the matching function. The hiring rate, h_t, is a convex function of J_t, for any value of $\alpha \in (0,1)$. For our linear wage function this means that it is also convex in z_t. Consequently, a rise in volatility then leads to an increase in expected values.

Tightness, θ_t, is also a convex function of z_t for any value of $\alpha \in (0,1)$. When J_t is small, for instance, an increase in J_t leads to small increases in vacancies. The reason is that small values of J_t are associated with low values of v_t. This implies a high marginal "productivity" of the matching function, so that small changes in the level of v_t are sufficient to restore the equilibrium conditions.

By contrast, f_t can either be a convex or a concave function of z_t depending on the value of α. Our results are based on $\alpha = 1/2$ in which case the job finding rate is linear in J_t and, thus, in z_t. This explains why the total volatility IRF for f_t is zero at all forecast horizons. The reason for the ambiguity and the dependence on the value of α is that the hiring rate is inversely related to J_t yet the job finding rate is inversely related to the hiring rate.

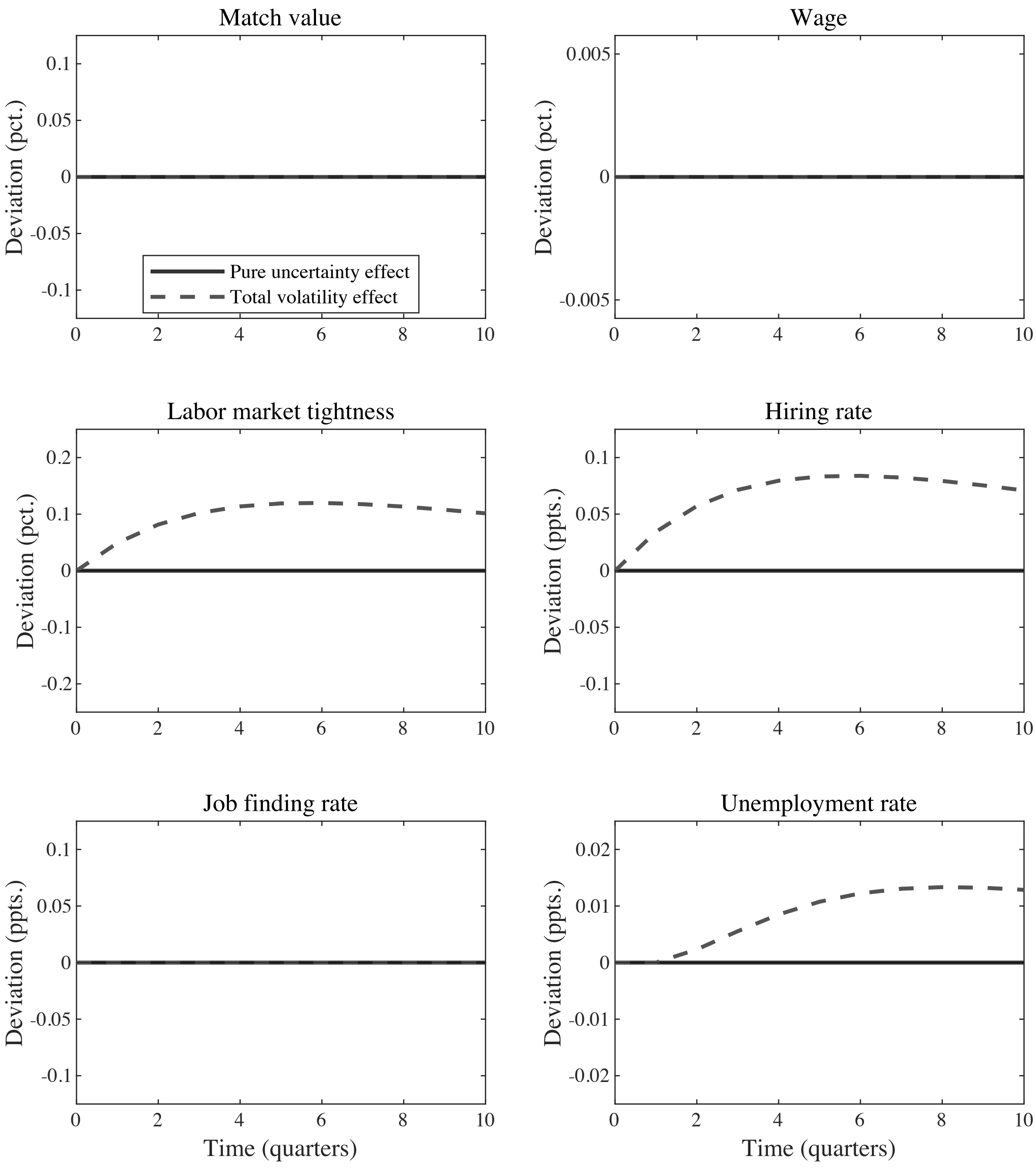
We now turn our attention to the effect of uncertainty on the employment rate, n_t. We repeat its law of motion for convenience.

$$n_t = (1-\delta)n_{t-1} + (1-(1-\delta)n_{t-1})f_t.$$

Although f_t always becomes more volatile, its expected value remains the same when $\alpha = 1/2$. But the total volatility IRFs indicate that this higher volatility is associated with a higher unemployment rate and, thus, lower employment rates. Why does an increase in the volatility of f_t reduce the expected future values of n_t? The reason is that the higher values of the job finding rate are expected to occur during expansions when fewer workers are searching for a job. Consequently, the impact on the employment rate will be smaller. By contrast, the lower values of the job finding rate will have a bigger impact because they are expected to occur during recessions when lots of workers are searching for a job.[17] In the period of the shock, the mass of searching workers, $1-(1-\delta)n_{t-1}$, is fixed and, hence, a higher volatility of f_t has no effect on expected employment. In the next few periods, this mass is still close to its steady-state value. But as time goes on, the asymmetric effect becomes more important when z_t shocks push unemployment either up or down. This explains the gradual increase for the unemployment IRF.

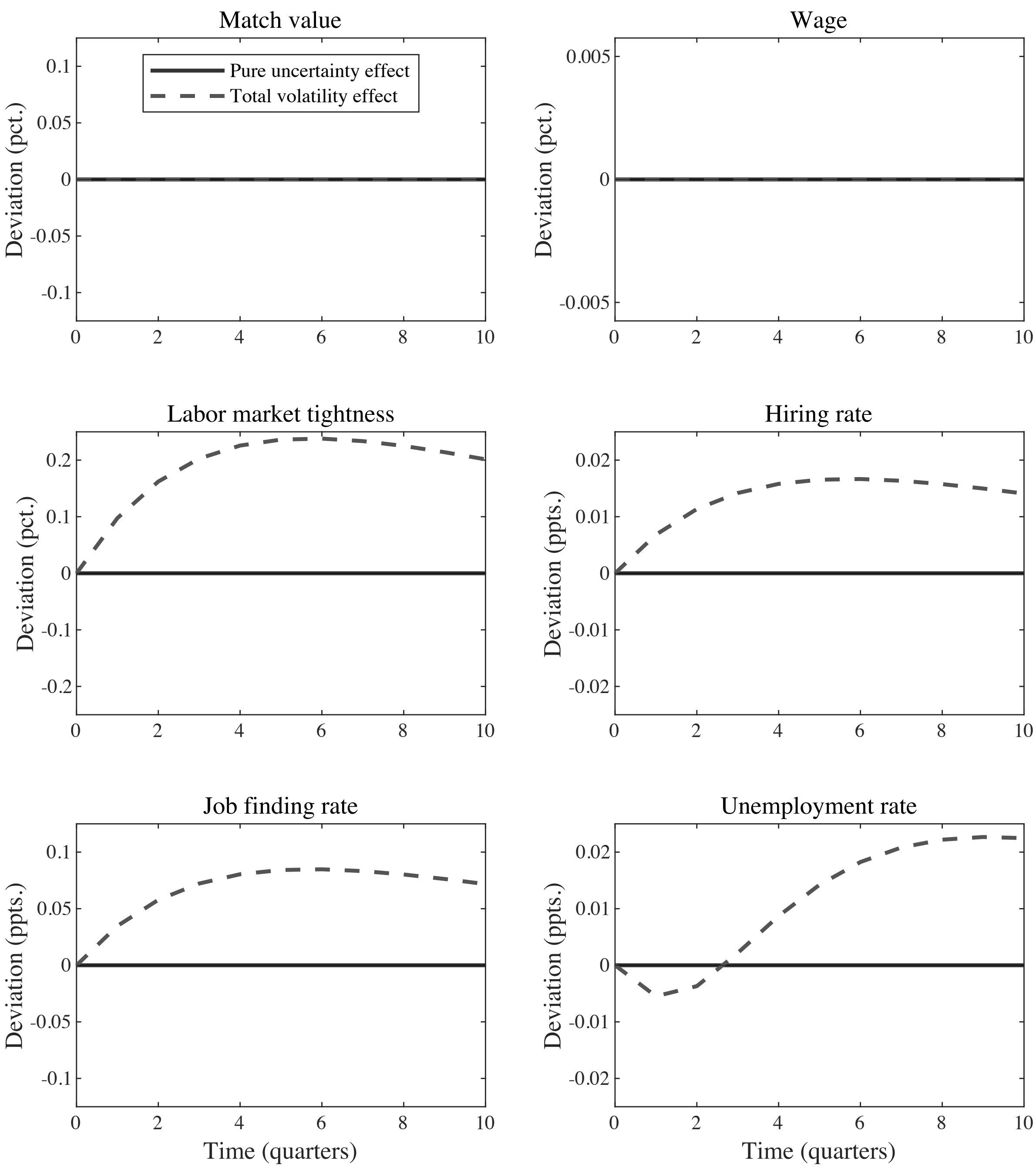
[16] The parameters of the version with linear wages are chosen to make it comparable to that with Nash bargaining. Specifically, the outside option χ is set such that the elasticity of labor market tightness with respect to productivity is unchanged relative to Nash bargaining. To this end, we exploit the close relationship between that elasticity and the fundamental surplus, $\overline{xz} - \chi$, as defined by Ljungqvist and Sargent (2017). Given the other

Figure 2: IRFs for uncertainty shock in standard SaM model with linear wage rule



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The "pure uncertainty" IRFs display how the economy responds when agents think volatility increases, but the higher volatility actually never materializes.

Figure 3: IRFs for uncertainty shock in standard SaM model with linear wage rule & low α



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The "pure uncertainty" IRFs display how the economy responds when agents think volatility increases, but the higher volatility actually never materializes. The value of α is equal to 0.2.

Why increased uncertainty might reduce unemployment due to matching frictions. When $\alpha = 1/2$, then the increased volatily has no effect on the average value of f_t. However, when $\alpha < 1/2$, then f_t is a convex function of z_t, which implies that the expected values of the job finding rate increases. Figure 3 plots the results when $\alpha = 0.2$. Since f_t is now a convex function of z_t, the period of higher volatility correspond to higher average job finding rates. Initially – as the unemployment rate is still close to its steady-state value – this pushes the unemployment rate down. This result illustrates that matching frictions by themselves can even lead to *decreases* in the unemployment rate during periods of heightened volatility, although the value of α has to be substantially lower than values typically assumed in the literature (cf. Petrongolo and Pissarides (2001)). Of course, an anticipated increase in volatility will still have no effect when $\alpha < 1/2$. This is another example that illustrates how the two types of higher volatility experiments generate quite different outcomes.

3.4. The non-trivial implications of Nash bargaining

As shown in section 3.1, an anticipated increase in uncertainty does lead to a reduction in the match value and a recession with Nash bargaining. So why are the results with Nash bargaining different from those with a linear wage rule? The answer actually follows quite directly from these results for the linear wage rule and the expression of the Nash-bargained wage rate, which we repeat here for convenience.

$$w_t^N = \omega \bar{x} z_t + (1-\omega)\chi + \omega\beta(1-\delta)E_t[\kappa\theta_{t+1}]$$

This expression makes clear that the wage does not only increase with the period-t benefits of not working, χ, and with current-period firm revenues, $\bar{x}z_t$. A higher expected value of future tightness likewise implies a higher wage rate this period.[18]

As discussed above, search frictions, and specifically the convexity of tightness, mean that the higher volatility in J_t increases the expected values of future tightness. With Nash bargaining, this expectation translates into higher current wage rates.[19] Higher current wages lead to a reduction in match value. The following proposition proves more formally that the match value J is concave in productivity under Nash bargaining.

Proposition 2. *Suppose that productivity is constant, $z_t = z_{t+1} = \cdots = z$, and wages are set by Nash bargaining, then $J(z)$ is a strictly concave function, and $\theta(z)$ is a strictly convex function.*

Proof. See online appendix OA. □

Intuitively, the free-entry condition together with the nonlinearity of the matching function ensures that θ is a convex function of J. Moreover, as the Nash bargained wage depends positively and linearly on tightness, the wage function is also convex in J. The concavity of J then follows from the convexity of the wage function.[20]

The concavity of $J(z_t)$ implies that its expected value should decrease if z_t becomes more volatile. But the story does not end here. The reduction in J_t leads to an immediate reduction in vacancy posting, which in turn puts an immediate *downward* effect on tightness and a reduction in the job finding rate. If one considers a period with an anticipated increase in volatility that never materializes, then the expected increase in tightness due to higher volatility

parameters, the bargaining weight ω is then pinned down by the steady-state version of equation (13). Parameter values are given in Table 1.

[17] See Hairault et al. (2010) and Jung and Kuester (2011).

[18] To gain intuition for why this is the case, notice that using the free-entry condition, we can write

$$\kappa\theta_{t+1} = \kappa\frac{v_{t+1}}{u_{t+1}^s} = \kappa\frac{f_{t+1}}{h_{t+1}} = f_{t+1}J_{t+1}.$$

That is, what matters for wage setting in terms of forward-looking behavior is the expected value of the product of next period's job finding rate and next period's firm value.

[19] In online appendix OB we show that if households are risk averse, then additional interaction effects come into play. In particular, whenever the marginal utility of consumption is elevated, this lowers the Nash-bargained wage rate.

[20] The above reasoning relies on the properties of J rather than productivity z. To complete the argument, notice that tightness will always be a convex function of z *unless* J is sufficiently concave. However, if this is the case, labor market tightness as well as wages are concave functions of z. Since w enters the match value negatively, this would imply that J must be convex, which is a contradiction. Thus J is concave in z. See section 3.3 for a discussion of a similar relationship between θ and z even when J is a linear function of z.

of J_t will never materialize either. Consequently, there is just downward pressure on firm value, tightness, and the wage rate, consistent with the IRFs given in figure 1. There will be an instantaneous jump down in these variables and a gradual return towards the (stochastic) steady state. What about the total volatility effect? For tightness we have the effect that works through the wage rate, that is, tightness drops because of the expected increase in the wage rate. This channel is strongest just after the shock and then leads to a monotonically declining effect. But we also have the effect arising from the nonlinearity of the matching function, which implies that tightness is a convex function. This latter feature gives rise to an upward effect on average tightness during periods of heightened volatility. The result is a non-monotone effect that is small at first.[21] Initially, the negative effect must dominate, but expected tightness becomes positive after two periods when it is overturned by the effect working through the nonlinearity of the matching function. The wage rate IRF leads the change in the expected value for tightness which follows directly from equation (2). The firm value is simply the mirror image of the wage rate since expected productivity actually does not change. Note that it must be the case that the total volatility IRF for tightness turns positive at some point. If it never did, then the wage response would not turn positive either, which means that firm value would not have dropped; but then tightness should not have fallen in the first place.

4. A search-and-matching model with option value

Section 3.2 showed that even though job creation is an irreversible investment, the standard search-and-matching (SaM) model does not have the other ingredient needed to generate an option-value channel – the mutual exclusivity of investment projects – since the choice to create a job this period does not restrict job creation in the future. In this section we propose an amended SaM model according to which elevated uncertainty does raise the value of waiting. Before specifying that model, we briefly discuss a simple experiment which demonstrates that an option-value channel is possible, in principle, simply by assuming that the mass of available entrepreneurs is *finite*. This example clarifies the crucial role of the free-entry condition in eliminating the option-value channel. It also serves as a stepping stone to understanding our proposed model.

Consider the model of section 2 with a linear wage rule. Productivity is constant and the economy starts out in steady state. We assume that the mass of entrepreneurs, while finite, is large enough for the steady state to be unaffected. In period t, the economy encounters the following increase in anticipated volatility. Aggregate productivity in some state of period $t+1$ is sufficiently great for the profits associated with posting a vacancy in that state to be strictly positive. In particular, there are simply not enough unmatched entrepreneurs available in the entire economy for these profits to be exhausted due to entry. That is, the free-entry condition no longer holds in that state, as

$$h_{t+1}J_{t+1} - \kappa > 0,$$

whereas it holds with equality in all other states.[22]

In period t, an idle entrepreneur is now faced with the choice of either posting a vacancy immediately, or waiting in the hope of entering when profits are strictly positive. Waiting is obviously a dominant strategy as long as profits in period t fall short of the expected profits in period $t+1$. Vacancies in period t therefore decline, and the hiring rate increases. This remains true until profits in period t are exactly equal to the expected profits of entry in period $t+1$. That is, until the arbitrage condition

$$h_t J_t - \kappa = E_t[h_{t+1}J_{t+1} - \kappa] > 0,$$

is satisfied. Thus, the (expectation of) positive profits available in period $t+1$, caused by a shortage of available entrepreneurs, gives rise to positive profits in period t; profits that are generated by a rise in the hiring rate, h_t, which is accomplished through a decline in vacancies. In short, a perceived increase in future uncertainty can give rise to an option value of waiting and a decline in economic activity in the present, if that increased volatility means that the constraint on the number of available entrepreneurs is expected to be binding in some future state of the world.

[21] If the shock occurs in period τ, then $z_{\tau+1}$ will be more volatile because $\varepsilon_{z,\tau+1}$ is more volatile. But $z_{\tau+2}$ will be more volatile because $\varepsilon_{z,\tau+2}$ *and* $z_{\tau+1}$ will be more volatile.

[22] Note, in particular, that the prospect of a large *fall* in aggregate productivity leaves expected profits unaffected, since such a shock lead to an increase in the slack of the constraint on the available number of entrepreneurs.

While this simple extension of the baseline model is sufficient to give rise to an option-value channel, is suffers from several disadvantages. Firstly, for an option-value mechanism to operate in this environment, one had to postulate the existence of states of the world in which there is literally nobody left to create jobs, regardless of how great the associated profits are. That seems implausible. More broadly, the distribution of shocks must be such that the free-entry condition is binding in some states but not in others. That is, the presence of option-value effects is sensitive to assumptions about the size of shocks. Finally, the requirement that the constraint on the number of entrepreneurs be occasionally binding complicates the numerical analysis.

4.1. A model with firm heterogeneity

So what can be done? Clearly, we have to maintain the assumption of a finite mass of entrepreneurs, lest free entry drive expected profits to zero in all states of the world, eliminating the possibility of an option-value channel. At the same time, it is desirable to have an internal solution. This can be accomplished by having heterogeneity in productivity among idle entrepreneurs. The simple modification gives rise to a framework in which there are *always* idle entrepreneurs, but only *some* that find it profitable to enter the matching market. The measure that finds it profitable to do so is endogenous and time-varying. At the same time, the option value of waiting remains operative, since higher uncertainty gives entrepreneurs upward potential, whereas they are shielded from downward risk, since they can always choose not to post vacancies.[23]

4.1.1. Setup

We assume that there is a finite, and constant, mass of potential entrepreneurs, Υ.[24] We adopt the standard convention in the SaM literature according to which each entrepreneur can potentially create a one-worker firm. In every period, each unmatched entrepreneur receives an *iid* productivity draw, a, from the cumulative distribution function, $F(a)$, with mean zero. The distribution is uniform on the interval $A = [-\overline{a}, \overline{a}]$, with $\overline{a} = \sqrt{3}\sigma_a$, where σ_a is the standard deviation of a.[25]

If an entrepreneur is successful in creating a new firm, the idiosyncratic productivity draw, a, is realized and lasts permanently throughout the match. As in the baseline model, the firm is then only dissolved by exogenous separation, which occurs at a rate δ. Entrepreneurs have finite lives, in that following separation the entrepreneur "dies" and gets replaced.[26] If, on the other hand, the entrepreneur is unsuccessful in creating a firm, she receives a new productivity draw in the subsequent period. As a consequence, only entrepreneurs with a high enough value for a will find it worthwhile to pay the cost of posting a vacancy. Others may instead find it more beneficial to wait for the opportunity of receiving a better draw in the future. That is, there is scope for an option value of waiting, without having to rely on there being states of the world in which there are no entrepreneurs left who conceivably could post further vacancies.[27]

With idiosyncratic productivity shocks the firm value is given by[28]

$$J_t(a) = (1-\omega)(\overline{x}(z_t + a) - \chi) + \beta(1-\delta)E_t[J_{t+1}(a)]. \tag{15}$$

[23]The models of Coles and Kelishomi (2018) and Leduc and Liu (2020b), referred to already in Footnote 4, likewise feature firm heterogeneity and an imperfectly elastic entry margin. Nonetheless, these models rule out an option-value channel by assumption. The reason is that in their setup, each one of a fixed mass of entrepreneurs can run several, independent business opportunities, and entering today does not preclude entering again next period. That is, mutual exclusivity does not obtain.

[24]Section OD.2 in the online appendix considers a more general version in which Υ is allowed to be time-varying. It nests both the model discussed here and the standard SaM model with free entry.

[25]Results with a Normal distribution are similar and are reported in online appendix OF.2.

[26]The match value does not feature a term related to the value of waiting as entrepreneurs die upon separation. This assumption is made to allow for a transparent analysis. When entrepreneurs live for ever, they would receive a new draw of a after separation and – just like entrepreneurs who wait – benefit from increased uncertainty although only at some future date. Online appendix OD.1 provides an alternative setup with infinitely-lived entrepreneurs.

[27]For the standard SaM model, there exists an equivalent representation with a representative n-worker firm. Online appendix OE derives such a representation for a generalized version of our model with idiosyncratic productivity dispersion. Therefore, both the standard and the alternative model can be interpreted as explaining changes in employment either through the extensive margin (changes in firm creation), the intensive margin (changes in firm size), or both.

[28]The notation is somewhat simplified in that it does not specify that a is the draw that the entrepreneur received in the period the match was created. Over time, firm level productivity, $z_t + a$, only varies with aggregate productivity, z_t. We also do not add a subscript to indicate that the level of a is firm-specific.

In the baseline model, the free-entry condition ensured that the value of an idle entrepreneur is always zero. In the current case, by contrast, the measure of entrepreneurs, Υ, is finite, and the value of being an idle entrepreneur prior to the revelation of the idiosyncratic draw, J_t^U, is non-negative and given by

$$J_t^U = \int_A \max\left\{\beta E_t[J_{t+1}^U], h_t J_t(a) + (1-h_t)\beta E_t[J_{t+1}^U] - \kappa\right\} dF(a) \tag{16}$$

$$= \int_A \max\left\{0, h_t(J_t(a) - \beta E_t[J_{t+1}^U]) - \kappa\right\} dF(a) + \beta E_t[J_{t+1}^U]. \tag{17}$$

Define $\hat{a}_t$ as the productivity cut-off that renders an entrepreneur indifferent between entering or not; that is,

$$h_t(J_t(\hat{a}_t) - \beta E_t[J_{t+1}^U]) - \kappa = 0. \tag{18}$$

Moreover, denote a_t^* as the expected value of a conditional on a being above the cutoff level, and p_t as the probability of such a draw. That is,

$$p_t = 1 - F(\hat{a}_t), a_t^* = \frac{1}{p_t}\int_{\hat{a}_t}^{\overline{a}} a dF(a) = E_t[a | a \geq \hat{a}_t]. \tag{19}$$

Then the value of an idle entrepreneur can be written as

$$J_t^U = p_t\left(h_t(J_t(a_t^*) - \beta E_t[J_{t+1}^U]) - \kappa\right) + \beta E_t[J_{t+1}^U]. \tag{20}$$

Lastly, the number of vacancies is given by

$$v_t = p_t(\Upsilon - (1-\delta)n_{t-1}). \tag{21}$$

Thus, in contrast to the previous framework, the firm value is now provided by equation (15), and the free-entry condition is replaced by equation (18); the equations for h_t, f_t, n_t, as well as the exogenous processes remain the same.

Before providing a qualitative analysis of the option-value channel it is necessary to touch upon some aspects of the calibration (see section 4.2 for additional details). In particular, our ambition is to keep the heterogenous-firm version as close as possible to the baseline, and for both frameworks to coincide – at least with respect to the key variables – at the steady state. In the baseline framework the cut-off level $\hat{a}$ is, by construction, zero. Thus, we calibrate the model such that the steady-state value of $\hat{a}$ remains at zero for any value of σ_a. Given the symmetry of the distribution this implies that $p = 0.5$.[29] Moreover, following equation (21), and imposing the steady-state values of vacancies, v, and employment, n, from the baseline model, one finds that Υ must be set as $\Upsilon = 2v + (1-\delta)n$. Another salient implication of this choice of $\hat{a}$ is that at the steady state, the measure of idle entrepreneurs posting a vacancies is equally large as that of idle entrepreneurs that are not. Thus, the constraint on the number of entrepreneurs is unlikely to be binding even for fairly large shocks and we verified that it indeed never is for any of the numerical exercises discussed.

4.1.2. An option value of waiting

The emergence of an option-value channel in this framework is intuitive and visible even in the absence of aggregate risk. We first explain how the channel emerges only due to idiosyncratic risk, and then discuss how a similar effect arises from aggregate volatility. In online appendix OC, we furthermore develop a two-period version of this model with heterogeneous productivity levels which is helpful in providing some graphical intuition as well as some analytical results.

Idiosyncratic risk. Suppose that there is no aggregate risk and that the cross sectional dispersion in productivity is zero; that is, $\sigma_a = 0$. Provided that there is a sufficient mass of available entrepreneurs to exhaust all (excess) profits of

[29] Online appendix OF.4 provides the results when this fraction is equal to 0.2 instead.

entry, the first term in equation (17) must equal zero. That is,

$$\max\left\{0, h(J(0)-\beta J^U)-\kappa\right\}=0, \tag{22}$$

where we dropped time subscripts given the absence of aggregate uncertainty. Consequently, $J^U=0$, and the above equation simply replicates the free-entry condition in the standard SaM model. Thus, with $\sigma_a=0$, the heterogeneous-firm model nests the baseline.

Suppose instead that $\sigma_a>0$. If $\hat{a}_t$ were unaffected by this alteration (remaining at zero), so would the hiring rate, h_t. By contrast, the presence of cross-sectional dispersion in productivity implies that a^* – i.e., the expected value of the idiosyncratic component conditional upon entry – must rise above zero. This means that the value of waiting, $\beta E_t[J^U_{t+1}]$, is positive as well. Consequently, an entrepreneur with $a=0$ now prefers to wait in the hope of getting a better draw next period. Consequently, $\hat{a}_t$ will increase until the hiring rate has dropped sufficiently so that the expected profits of vacancy posting at the new cut-off level equals the value of waiting.[30]

Would it not be possible for changes in the hiring rate to drive the expected value of waiting to zero? No. If that were true, then the expected profits of vacancy-posting would be equal to zero for an entrepreneur with $a=\hat{a}_t$. With idiosyncratic dispersion, however, this agent has some probability of receiving a draw for a in the future that exceeds $\hat{a}_t$ in which case the expected profits must be strictly positive.[31] The more cross-sectional dispersion, as indicated by σ_a, the larger the difference between $\hat{a}$ and a^*; that is, the stronger the option value of waiting due to idiosyncratic risk, the lower p_t, and the higher the unemployment rate. The leftmost graph in figure 4 illustrates this relationship between σ_a and the steady state level of the unemployment rate. As can be seen, the mechanism is powerful; an increase in the standard deviation of a from zero to 0.01 (that is, one percent of the output level without idiosyncratic dispersion) increases the steady-state unemployment rate from 6.4% to almost 14%.

Aggregate risk. The presence of aggregate risk also gives rise to an option value of waiting mechanism, which operates similarly, but not identically nor independently, to the above mechanism. To understand the nuance, notice that a higher value for z_{t+1} would increase the value of $J_{t+1}(a)$, while a lower value for z_{t+1} would result in a decline. When wages are linear in productivity and entrepreneurs die after an exogenous separation, which is the case in this framework, the increase and decrease in J_{t+1} exactly offset each other. Nevertheless there still is an option value of waiting. The reason is as follows. The increase in z_{t+1} generally leads to a reduction in the cutoff value $\hat{a}_{t+1}$ (and, hence, in a^*_{t+1}), since *total* productivity, $z_{t+1}+\hat{a}_{t+1}$, will anyway increase. Similarly, the decrease in z_{t+1} generally leads to an increase in $\hat{a}_{t+1}$ (and a^*_{t+1}). Consequently, the probability of entering and thereby benefiting from an increase in J_{t+1} is higher than that of the decrease.[32] Therefore, an anticipated increase in future aggregate volatility increases the conditional expected value of a match, $J_{t+1}(a^*_{t+1})$, which thereby raises the value of waiting, J^U_{t+1}; the option-value channel materializes.

It ought to be noted that the presence of cross-section dispersion – alongside, of course, the finite measure of entrepreneurs – is necessary for this mechanism to operate *at all*.[33] Indeed, entrepreneurs can only "benefit" from a higher match value *if* the profits of entry can be positive. Absent cross-sectional dispersion (and with a potentially infinite measure of entrepreneurs), the hiring rate would otherwise adjust to ensure that the expected profits of entry are zero in all time periods and in all states of the world, and the option-value channel would close down. The rightmost graph in figure 4 shows the relationship between the amount of cross-sectional dispersion, σ_a, and the steady-state elasticity of labor market tightness with respect to aggregate productivity. Less dispersion implies a higher elasticity, which reflects the fact that dispersion dampens the movements in the hiring rate.

4.2. Recalibration scheme

An insight from the previous section is that the amount of cross-sectional dispersion, σ_a, alters some of the key properties of the model. In particular, a higher value of σ_a is associated with a higher steady-state unemployment

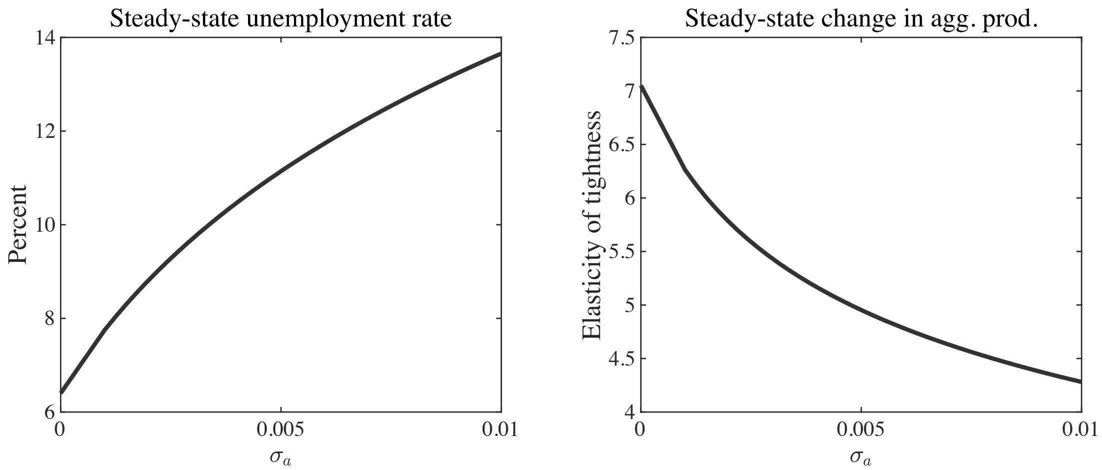
[30] It is of course essential that entrepreneurs have the option to *not* post a vacancy in the future. That is, expected profits are always bounded below at zero. This leads to a convex payoff function and Jensen's inequality then implies that uncertainty raises expected values.

[31] With idiosyncratic dispersion, a^*_t could be equal to $\hat{a}_t$, but only if $\hat{a}_t$ is at the upper bound of the distribution, that is, when nobody would want to post vacancies. This does not happen for any of our parameterizations, because changes in matching probabilities always ensure an interior solution for $\hat{a}_t$.

[32] Phrased in another way, the higher J_{t+1} is multiplied by a higher value of p_{t+1} than the lower J_{t+1}. See online appendix OC for an intuitive and graphical exposition using a 2-period version of the model.

[33] That is, unless we rely on a shock distribution that render the free-entry condition occasionally binding.

Figure 4: Cross-sectional dispersion and steady-state properties



Notes: The panels display key moment properties as a function of the amount of cross-sectional dispersion. The left panel indicates the steady-state unemployment rate, while the right panel plots the steady-state elasticity of labor market tightness with respect to aggregate productivity. All other model parameters are kept fixed and are equal to the ones given in table 1 for the case with the linear wage rule and no cross-sectional dispersion.

rate for a given value of aggregate productivity. Yet, a key element of the calibration strategy of Leduc and Liu (2016) and adopted here is that the theoretical steady-state unemployment rate matches its empirical counterpart. In addition, the volatility of the hiring rate is declining in σ_a. In view of this, we pursue a recalibration strategy which ensures that *irrespective* of the chosen value of σ_a, the model economy matches key empirical targets and features a comparable degree of aggregate volatility to the baseline model of section 2. Specifically, (i) the steady-state values of all endogenous variables are unchanged; and (ii) the steady-state elasticity of labor market tightness with respect to aggregate productivity equals the baseline.

The key parameter to obtain the latter target is χ, which controls the value of the worker's outside option during bargaining. By choosing larger values for χ when σ_a is higher, we reduce the contemporaneous surplus, $\bar{x}(z_t + a - \chi)$, which renders the model variables more volatile – offsetting the lower volatility implied by a wider cross-sectional distribution. Next, for the steady-state rate of unemployment to be the same as in the baseline model, the steady-state value of the cutoff level $\hat{a}$ must be equal to zero for the different values of σ_a considered.[34] The key parameter to accomplish this is the worker bargaining power, ω. When χ is increased, the share that accrues to the entrepreneur, i.e., $1-\omega$, must increase to ensure the same level of steady-state vacancy posting.

Lastly, the steady-state total productivity of the *average* firm is given by $\bar{z} + a^*$. Since a^* increases with σ_a, we adjust the value of $\bar{z}$ downward to compensate for this effect. A benefit of this approach is that the steady-state value of $\left(J(\hat{a}) - \beta J^U\right)$, i.e., the difference in the value of a match at the cutoff relative to the value of an unmatched entrepreneur, is the same across economies.[35] Since that term plays a key role in driving the dynamics of the model, this aspect of the recalibration procedure assists with the interpretation of the results.

Our recalibration scheme imposes a natural range for the values of σ_a. As σ_a increases, we need to increase χ and lower ω. Above a value of $\sigma_a = 0.003$, ω quickly approaches its natural lower bound of 0. As this is a fairly low value, we adopt it as a benchmark.[36] Following our recalibration procedure, when $\sigma_a = 0.003$, we set $\chi = 0.757$, $\omega = 0.636$, and $\bar{z} = 0.997$. The remaining parameters are unchanged and available in table 1, while the mass of entrepreneurs Υ given $p = 0.5$ is equal to 1.11.

[34] Recall that Υ is set such that the fraction of entrepreneurs that enters the matching market is $1/2$ in the economy without cross-sectional dispersion.

[35] The calibration strategy involves setting the fraction of output spent on vacancy posting costs in steady-state, $\kappa v/(\bar{z}a^* n)$, equal to 2%. The adjustment of $\bar{z}$ ensures that κ is the same across economies, which together with the fact that h is calibrated to be the same across economies means that $J(\hat{a}) - \beta J^U$ is the same across economies.

[36] This amount of idiosyncratic dispersion is small relative to the degree of cross-sectional productivity dispersion observed in the real world. See, for example, Sterk et al. (2020). It is not surprising that our framework with ex-ante identical entrepreneurs cannot generate the observed differences which are likely to arise from numerous factors besides those present in a simplified model as we consider here.

4.3. Numerical results

Figure 5 plots the IRFs for a volatility shock given $\sigma_a = 0.003$.[37] The following observations stand out. First, and consistent with the preceding qualitative discussion, an increase in anticipated aggregate uncertainty causes a recession even though wages are linear in productivity. The anticipation of heightened future volatility increases the value of waiting, which in turn reduces entry and vacancy-posting, lowering the job finding rate and, ultimately, pushing up the unemployment rate. Second, the total volatility effects are much larger than the pure uncertainty effects (consistent with the empirical findings of Berger et al. (2020)). This result strengthens our recommendation to consider both types of IRFs when studying the implications of time-varying volatility. Indeed, the total volatility IRFs strongly resemble those obtained in the absence of firm heterogeneity, and for similar reasons; the nonlinearities of the matching function generate a persistent rise in both the unemployment rate and the hiring rate, as discussed in section 3.

A few subtleties are worth pointing out. For one, in the presence of idiosyncratic dispersion, aggregate output is no longer proportional to $z_t n_t$. The composition of the sample of producing firms matters, as they vary in their individual productivity levels. Specifically, changes in the number of vacancies posted occur through changes in the cutoff level, which in turn affects the average productivity of producing firms. Following a volatility shock, the value of waiting rises on impact due to anticipation effects. The associated *increase* in the average productivity level of those firms that do enter dampens, but does not overturn, the reduction in output due to the fall in employment – an effect that is absent in the model with homogeneous entrepreneurs.[38] However, in the case of total volatility effects, this dampening effect is short-lived. The sharp rise in the unemployment rate and the associated increase in vacancy posting (through an increased entry probability) takes hold, whereupon the average productivity of entrants declines. Consequently, output not only falls because of the decline in employment, but also due to composition effects.

Moreover, uncertainty shocks have non-zero effects on the job finding rate. This result stands in contrast to the baseline model (with linear wages), according to which both the pure and the total volatility effects on the job finding rate are equal to zero in expectation when the matching elasticity, α, is equal to 0.5. Here, instead, the presence of a wait-and-see mechanism – specifically the associated reduction in the entry probability – causes the job-finding rate to decline when perceived uncertainty rises. The total volatility effect on the job-finding rate is likewise negative, larger, and more persistent. To see why, recall from the discussion in section 3, that the hiring rate is a convex function of z_t. Equation (18) makes clear that the observed increase in the value of an unmatched entrepreneur introduces an additional positive effect on the hiring rate, and thus a negative effect on the average job finding rate.

Quantitative comparison. To evaluate the quantitative impact of uncertainty shocks in the current framework, we compare our results with those described in section 3.1 for the standard SaM model with free entry and Nash bargaining (recall that this wage-setting assumption is the key reason why pure uncertainty shocks have non-zero effects in that model).

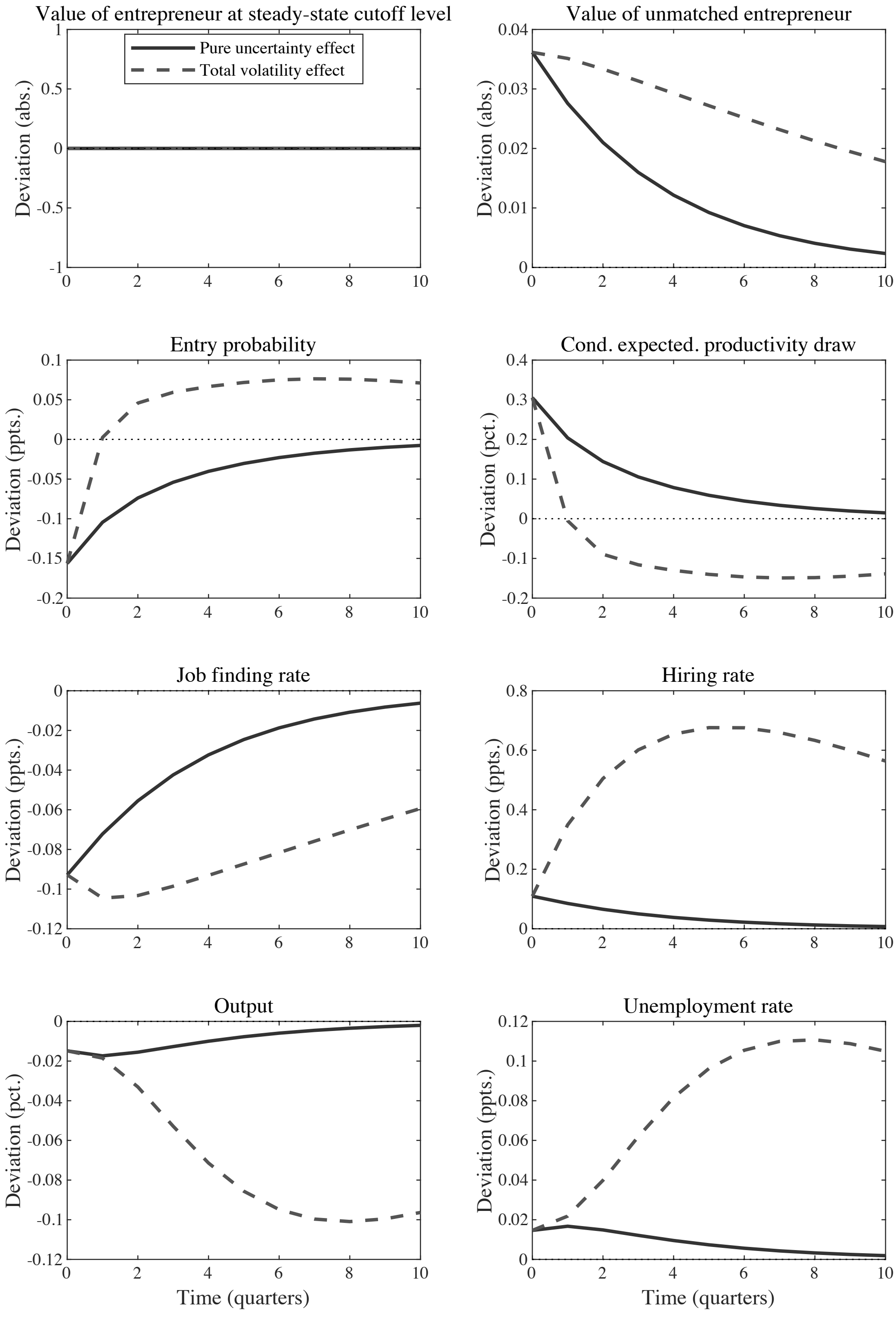
Figure 6 illustrates the total volatility effect of an uncertainty shock on the unemployment rate, both on impact (left graph) and at the maximum along the IRF (right graph). The effect on impact is entirely due to anticipation, and the maximum total volatility effect occurs after roughly eight quarters. The horizontal lines in the two graphs indicate, for comparison purposes, the same statistics obtained in the baseline model with Nash bargaining. Recall that in the model with heterogeneity we adopted the linear wage rule and deliberately chose the mass of entrepreneurs, Υ, such that the constraint on their number is never binding. Hence, with barely any cross-sectional dispersion, there should be no quantitatively significant anticipation effects due to a volatility shock. The figure reveals that, indeed, for very small values of σ_a, we are essentially back to the model of section 2 with linear wages.

Nonetheless, even with still relatively little cross-sectional dispersion, volatility shocks can generate a substantial effect on unemployment. Thus, a value of σ_a equal to 0.003 implies that the entrepreneur with the most productive draw for a is just one percent more productive than the entrepreneur with the least productive draw. In spite of that, both the initial pure uncertainty effect as well as the maximum total volatility effect are more than double than what is generated in the baseline model with Nash bargaining.

[37] See online appendix OF.3 for the same set of IRFs given $\sigma_a = 0.001$. Additionally, figure 6 plots the impact and maximum total volatility effect on the unemployment rate, specifically, as a function of σ_a.

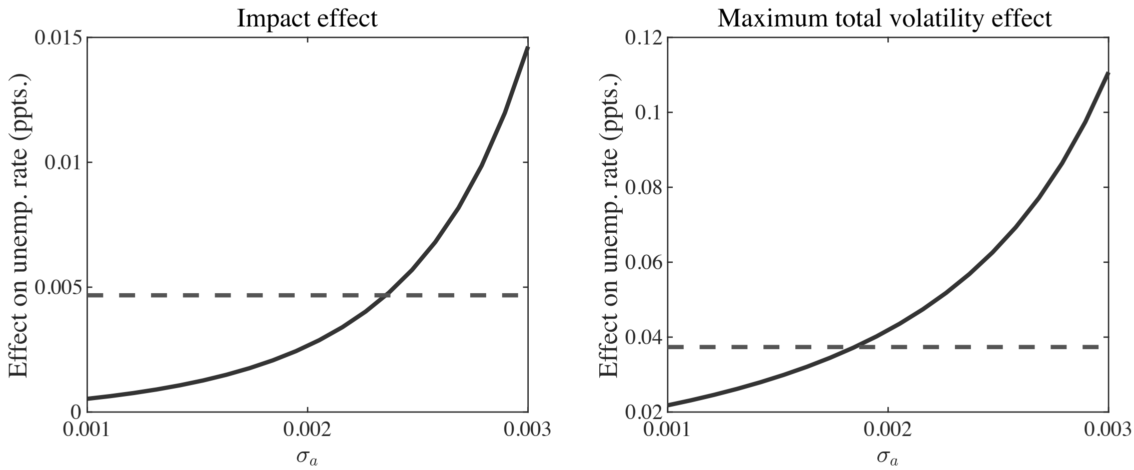
[38] Figure OF.3 in the online appendix illustrates that this effect can, in principle, lead to a small initial increase in output when $\sigma_a = 0.001$.

Figure 5: IRFs for uncertainty shock in SaM model with cross-sectional dispersion; $\sigma_a = 0.003$



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The "pure uncertainty" IRFs display how the economy responds when agents think volatility will increase, but the higher volatility actually never materializes.

Figure 6: Cross-sectional dispersion and aggregate uncertainty effects



Notes: The panels display the initial impact and the maximum total volatility impact of a unit-increase in $\varepsilon_{\sigma,t}$ as a function of the amount of cross-sectional dispersion, σ_a. Other model parameters are recalibrated to make the economies with different values of σ_a comparable.

Robustness checks. In online appendix OF, we discuss the results of several robustness exercises. Most importantly, our baseline specification of the model assumes that an entrepreneur can post only one vacancy and then creates a job with probability h_t ("stochastic hiring"). An alternative is to suppose that the entrepreneur posts $1/h_t$ vacancies and then creates one job with certainty ("non-stochastic hiring"). In the standard SaM model with risk-neutral entrepreneurs, these two options generate the exact same model properties. In our modified framework, entrepreneurs are also risk neutral and the two different specifications imply the same qualitative properties. Quantitatively, however, when there is both aggregate and idiosyncratic uncertainty, a model with non-stochastic hiring generates a substantially stronger option-value effect due to elevated volatility than implied by our baseline specification.

5. Concluding remarks

The option value of waiting to invest in the presence of uncertainty strikes many as a plausible mechanism to rationalize the empirical finding that elevated uncertainty negatively impacts economic activity. Moreover, the popularity of the search-and-matching (SaM) literature underscores the usefulness of modeling job creation as an investment. Yet, we showed that the usual assumption in that literature of there being a "potentially infinite number" of entrepreneurs to take advantage of opportunities in the matching market eliminates any grounds for wait-and-see behavior. The standard SaM model, therefore, cannot be used to rationalize the effects of uncertainty shocks in terms of an option-value channel. If, on the other hand, there is a limit on the number of potential entrepreneurs and they vary in their idiosyncratic productivity levels – two modifications that are both plausible and can be introduced into the model in a tractable manner – the model properties completely change. In particular, an increase in perceived volatility then does indeed robustly increase the option value of waiting, causing a reduction in job creation and higher unemployment.

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Online Appendix for:
"Volatile Hiring: Uncertainty in Search and Matching Models"*

Wouter Den Haan
London School of Economics, CEPR and CFM

Lukas B. Freund
University of Cambridge

Pontus Rendahl
Copenhagen Business School, CEPR and CFM

July 2021

This appendix contains supplemental material for the article "Volatile Hiring: Uncertainty in Search and Matching Models."

Any references to equations, figures, tables or sections that are not preceded by a capital letter refer to the main article.

* *E-mail*: Den Haan: w.denhaan@lse.ac.uk; Freund: lukas.beat.freund@gmail.com; Rendahl: pontus.rendahl@gmail.com.

Appendix OA Derivations and proofs

OA.1 Derivation of Nash-bargained wage

Let V_t and U_t denote the value of an employed and an unemployed worker, respectively. That is,

$$V_t = w_t + \beta E_t \left[(1-\delta+\delta f_{t+1}) V_{t+1} + \delta (1-f_{t+1}) U_{t+1} \right],$$
$$U_t = \chi + \beta E_t \left[f_{t+1} V_{t+1} + (1-f_{t+1}) U_{t+1} \right].$$

Thus, the surplus to the household of accepting a job is given by

$$S_t = V_t - U_t = w_t - \chi + \beta E_t \left[(1-\delta)(1-f_{t+1}) S_{t+1} \right]. \tag{OA.1}$$

Similarly, the surplus to the firm of hiring a worker is simply the firm value, J_t, (which is repeated for convenience)

$$J_t = \bar{x} z_t - w_t + \beta (1-\delta) E_t [J_{t+1}]. \tag{OA.2}$$

Nash bargaining sets the wage, w_t, to maximize the Nash product such that

$$w_t = \arg\max \{ J_t^{1-\omega} S_t^{\omega} \},$$

where ω represents the bargaining power of the worker. The first order condition is given by

$$(1-\omega) S_t = J_t \omega. \tag{OA.3}$$

Using the first order condition in equation (OA.3) together with equations (OA.1) and (OA.2) gives

$$(1-\omega)(w_t-\chi)+\beta(1-\delta)\omega E_t[(1-f_{t+1})J_{t+1}]=\omega(\bar{x}z_t-w_t)+\beta(1-\delta)\omega E_t[J_{t+1}].$$

Solving this equation for w_t gives the following expression for the Nash-bargained wage rate:

$$w_t^N=\omega\bar{x}z_t+(1-\omega)\chi+\omega\beta(1-\delta)E_t[f_{t+1}J_{t+1}].$$

Using the free-entry condition, $\kappa=h_tJ_t$, and the relationship between the hiring rate, the job finding rate, and tightness, $f_t=h_t\theta_t$, we get the expression which relates the wage rate to expected tightness.

$$w_t^N=\omega\bar{x}z_t+(1-\omega)\chi+\omega\beta(1-\delta)\kappa E_t[\theta_{t+1}].$$

OA.2 Proof of Proposition 1

To derive Proposition 1, we substitute the linear wage rule given in equation (13) into the firm value equation in (1) and iterate forward.[OA.1] Thus,

$$\begin{aligned}J_t&=\bar{x}z_t-w_t+\beta(1-\delta)E_tJ_{t+1}\\&=E_t\Sigma_{j=0}^{\infty}\beta^j(1-\delta)^j(1-\omega)(\bar{x}z_{t+j}-\chi).\end{aligned}$$

[OA.1] We rule out exploding paths, such that

$$\lim_{j\to\infty}[\beta(1-\delta)]^jE_t[J_{t+j}]=0,\quad t=0,1,\ldots$$

Next, use the law of motion for productivity (5).

$$J_t = -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} + (1-\omega)\bar{x}z_t + \beta(1-\delta)(1-\omega)[\bar{x}((1-\rho_z)+\rho_z z_t)]$$
$$+\beta^2(1-\delta)^2(1-\omega)[\bar{x}((1-\rho_z)+\rho_z(1-\rho_z)+\rho_z^2 z_t)]$$
$$+\beta^3(1-\delta)^3(1-\omega)[\bar{x}((1-\rho_z)+\rho_z(1-\rho_z)+\rho_z^2(1-\rho_z)+\rho_z^3 z_t)]+\dots$$

Simplifying,

$$J_t = -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} + \frac{(1-\omega)\bar{x}z_t}{1-\beta(1-\delta)\rho_z} + \frac{\frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{1-\beta(1-\delta)}}{1-\beta(1-\delta)\rho_z},$$

and collect terms,

$$J_t = -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} + \frac{(1-\omega)\bar{x}z_t}{1-\beta(1-\delta)\rho_z} + \frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{(1-\beta(1-\delta))(1-\beta(1-\delta)\rho_z)}.$$

This final line corresponds to equation (14). □

OA.3 Proof of Proposition 2

The firm value is in this case given by

$$J(z) = \frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa\theta(z)}{1-\beta(1-\delta)}.$$

Suppose that $J(z)$ is (weakly) convex in the vicinity of some $z>0$. That is

$$tJ(z_1)+(1-t)J(z_2) \geq J(z),$$

for some $z_1>0$ and $z_2>0$ and any $t\in(0,1)$ such that $z = tz_1+(1-t)z_2$. Then by definition

$$\frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa(t\theta(z_1)+(1-t)\theta(z_2))}{1-\beta(1-\delta)} \geq \frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa\theta(z)}{1-\beta(1-\delta)},$$

or simply

$$(t\theta(z_1) + (1-t)\theta(z_2)) \leq \theta(z).$$

That is, $\theta(z)$ must be weakly concave in the vicinity of z.

The free-entry condition implies that

$$\theta(z) = \left(\frac{\psi}{\kappa} J(z)\right)^{\frac{1}{\alpha}},$$

which implies that $\theta(z)$ is a strictly convex function in the vicinity of z. As this is a contradiction, $J(z)$ must be strictly concave for all $z > 0$, which implies that $\theta(z)$ must be strictly convex for all $z > 0$. □

Appendix OB Risk aversion in the standard model

Our main analysis assumes that the representative household is risk neutral. This assumption carried the benefit of isolating the analysis to option-value considerations. Allowing for risk aversion introduces a number of complexities in the form of additional transmission channels and interaction effects. In this section, we give an indication of what direction they go. We highlight, in particular, that risk aversion alters the predictions of the canonical search-and-matching (SaM) model for the effects of uncertainty shocks on economic activity in two primary ways. First, the interaction of investor risk aversion and search-frictions in the pricing of firm equity gives rise to non-zero, adverse, pure uncertainty effects; even when the wage function is linear. This stands in marked contrast to the risk-neutral benchmark. Second, if households are risk averse, regular Nash bargaining over wages may dampen this uncertainty-induced recession.

To derive these results, we proceed in two steps. We start by assuming that wages are a linear function of current productivity, as in equation (13). And later we will add Nash bargaining to the analysis. In the main text, which assumed risk neutrality, we emphasized that under this specification, the stream of expected dividends from a match is unaffected by an increase in uncertainty (cf. Proposition 1). But risk aversion makes the representative household value any given dividend stream differently when uncertainty increases. The equation pinning down the period-t firm value incorporates stochastic discounting of the continuation value in the expectation term is

$$J_t = \bar{x}z_t - w_t + (1-\delta)E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}J_{t+1}\right]. \tag{OB.1}$$

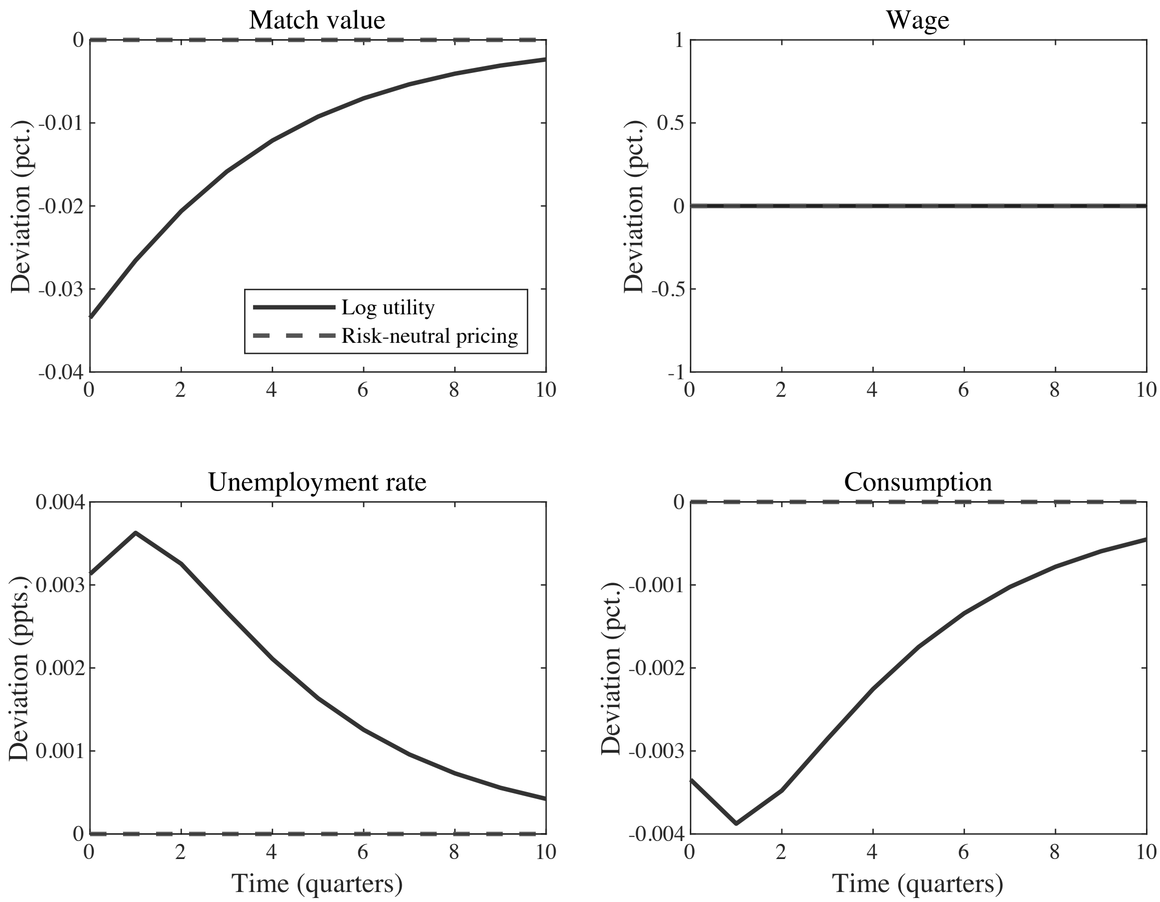
Moreover, as in Leduc and Liu (2016), households can save not only in form of equity, but also risk-free government bonds that pay a gross real interest rate R_t. As such, the system of equations is augmented by a standard bond Euler equation.[OB.1]

Freund and Rendahl (2020) explore this model with linear wages in depth, considering also

[OB.1] However, as prices are flexible, this augmentation merely prices bonds, and does not alter any economic quantities.

the role of nominal rigidities and shedding light on the simultaneous operation of supply and demand channels through which uncertainty shocks then affect economic activity. Here we briefly summarize their results for the transmission of uncertainty shocks in a setting with flexible prices.

Figure OB.1: Pure uncertainty IRFs in SaM model with risk aversion and linear wage rule



Notes: The figure shows the "pure uncertainty" IRFs to a unit-increase in $\varepsilon_{\sigma,t}$ for the SaM model with a linear wage rule and allowing for risk aversion. These IRFs display how the economy responds when agents think volatility will increase, but the higher volatility actually never materializes.

The solid line in figure OB.1 indicates the pure uncertainty IRF for the firm value, wages, unemployment, and consumption, assuming a logarithmic utility function. As can be seen, the increase in uncertainty causes the firm value to fall. A lower firm value means that the incentives to post vacancies are weakened, and the job-finding rate declines. Unemployment consequently rises, while output and consumption contract. Thus, while search frictions *by themselves* are insufficient to raise the unemployment in response to an increase in perceived uncertainty – as visualized by the the dashed line, which describes the pure uncertainty effect in the risk-neutral case – their *interaction with risk aversion* means that a rise in uncertainty lowers economic activity even when prices are flexible and the wage function is linear.

Three distinct mechanisms underpin this result. Two of them are expansionary, but they are dominated by a contractionary force which operates through the risk premium.[OB.2] On the one hand, two mechanisms trigger a rise in households' desire to save that leads them to value all assets, including the risky equity of intermediate goods firms, more highly when faced with a more uncertain future. The first mechanism is linked to the usual prudence motive associated with the marginal utility of consumption being convex; by Jensen's inequality, the perception of greater future volatility pushes up $E_t[u'(c_{t+1})]$. The starting point for the second expansionary mechanism is that, as described in section 3.3, search frictions mean that average unemployment is higher in periods of heightened volatility. Households *anticipating* the future to be more volatile, therefore, also *expect* average unemployment to be higher. If households aim to smooth consumption over time, this expectation reinforces their desire to save in form of both bonds and equity rather than consume in the present. At the same time, however, increased uncertainty about the future generates a stronger negative comovement between the marginal utility of consumption and the equity value; low payoffs are expected for precisely those periods where consumption will be low and, hence, when dividend income would be more valuable (and vice versa). This negative comovement is captured by a rise in the required risk premium, causing a fall in the firm value. In summary, therefore, a rise in uncertainty lowers economic activity when households are risk averse and when the wage is unresponsive to expected movements in either labor market tightness or marginal utility. The reason is that households anticipating greater future volatility require a larger risk premium to compensate them for holding the equity of firms with long-term employment relationships. More costly equity acts to suppress hiring activity.

[OB.2] In principle, the net effect of these mechanisms is *ex ante* ambiguous. Freund and Rendahl (2020) underscore that the risk premium can be large and volatile in the SaM setting, the reason being that hiring a worker is akin to investing in risky assets with long-duration payoffs.

Next, suppose that wages are not a linear function of productivity but, instead, are determined by Nash bargaining. As such, the theoretical environment we consider here corresponds to the flexible-price version of Leduc and Liu (2016, see their section 4.2.1).[OB.3] We follow Leduc and Liu (2016) in supposing that the worker's reservation value consists of a combination of unemployment benefits, ϕ, and disutility of supplying labor, χ (or, equivalently, a linear utility parameter for leisure).[OB.4] Extending equation (2), the wage is pinned down as

$$w_t^N = (1-\omega)\left(\phi + \frac{\chi}{u'(c_t)}\right) + \omega\left(\bar{x}z_t + \beta(1-\delta)\kappa E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\frac{v_{t+1}}{u^s_{t+1}}\right]\right), \tag{OB.2}$$

Relative to the risk-neutral Nash wage, household risk aversion shows up in two ways in equation (OB.2). First, the utility of leisure accrues to the worker's surplus when bargaining, but in consumption units (which is then suitably shared between the firm and the worker according to their respective bargaining power). Thus, the parameter appears in the wage equation divided by the marginal utility of consumption. Second, next-period labor market tightness is discounted with the marginal rate of substitution.

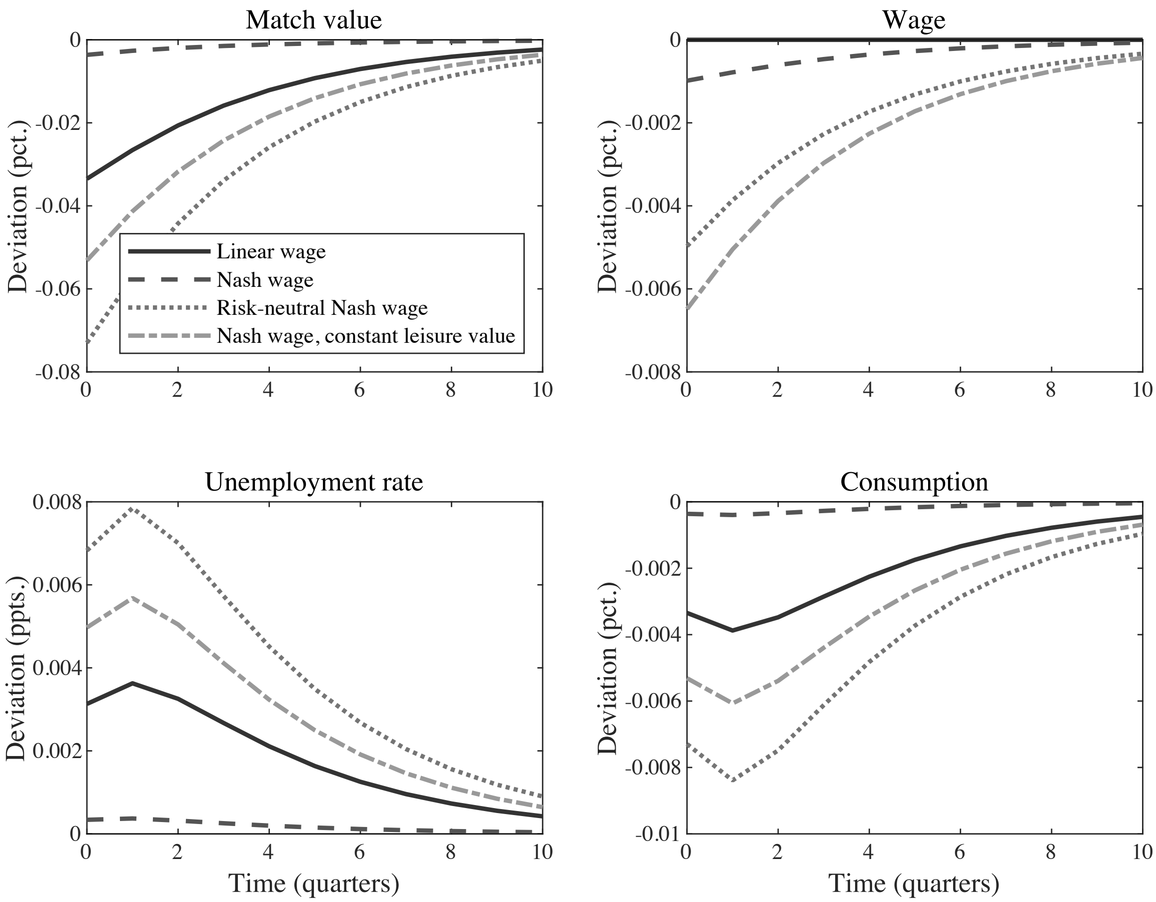
Figure OB.2 reports the effects of an increase in anticipated volatility under four wage-setting specifications: (a.) the benchmark case with linear wages (solid line); (b.) "regular" Nash bargaining (following equation (OB.2), dashed line); (c.) the wage materializing under Nash bargaining with risk neutrality (following equation (2); dashed-dotted line); and (d.) the wage materializing under Nash bargaining when the money-metric value of the utility of leisure is held at its steady-state value (dotted line). For (c.), we only assume risk neutrality in deriving the wage equation.

The figure makes clear that Nash bargaining can either dampen ("regular") or magnify (risk-neutral Nash wages) the IRFs. The difference between (a.) and (c.) is essentially the "Nash-wage

[OB.3] Different from Leduc and Liu (2016), we abstract from extrinsic wage rigidity to preserve greater transparency. On the same grounds, we also continue to assume that resources expended on vacancy posting are rebated to the household. Quantitatively, the numbers we report for the case of Nash bargaining are therefore most comparable, but do not exactly coincide with, the results reported in their online appendix, specifically the dotted line in their Figure A6.

[OB.4] As far as the calibration of the model is concerned, we also stay as close as possible to Leduc and Liu (2016). As such, under Nash bargaining we take the flow benefits of unemployment, ϕ, to be equal to 0.25, while the disutility of working, χ, is set equal to 0.5348. Given a steady-state value of consumption $\bar{c} = \bar{n} = 0.9360$, the monetary value of $(\phi + \chi/u'(c_t))$ in steady-state monetary terms is therefore 0.751, just as in the risk-neutral setting. The steady-state elasticity of labor market tightness with respect to productivity thus remains unchanged.

Figure OB.2: Pure uncertainty IRFs in SaM model with risk aversion



Notes: The figure shows the "pure uncertainty" IRFs to a unit-increase in $\varepsilon_{\sigma,t}$ for the SaM model, allowing for risk aversion and considering alternative wage specifications.

channel" explored in the main text, that is, the interaction of higher expected future labor market tightness and Nash bargaining adds to the recessionary effect of a rise in the risk premium.

The possibility of a dampening effect, on the other hand, is due to the fact that whenever the household's marginal utility of consumption is elevated relative to the steady state, this lowers the wage rate. For when a recession materializes, consumption falls and the marginal utility of consumption increases. Thus, the monetary value of leisure declines, which, ceteris paribus, means that the wage is permitted to fall more than under risk neutrality. With more wage flexibility, there are smaller movements in asset prices, and the contraction is less pronounced (which, seemingly contradictory, leads to a smaller decline in wages). What is more, when uncertainty is *expected* to be high, the same channels that give rise to a precautionary motive raise the expected marginal utility of consumption. By the preceding logic, this mechanism lowers the expected wage, and, hence, raises expected future dividends. It therefore exerts upward pressure on the firm value, J_t, which, after all, is simply the present discounted value of dividends. Because J_t is elevated relative to the risk-neutral benchmark, vacancy creation and employment are boosted. As a result, consumption is actually higher and the marginal utility consequently lower, which raises the realized wage, other things equal.[OB.5] The dotted line in Figure OB.2 (case d.) shows this by adding a Nash bargaining solution which holds the utility component of leisure fixed in money terms over the cycle. Compared to the baseline with Nash wages, the effect of a rise in uncertainty is worse than under regular Nash bargaining or, indeed, the linear wage specification.

A final nuance arises from the fact that under regular Nash bargaining with risk aversion, the stochastic discount factor is involved in determining the worker's surplus. In terms of equation (OB.2), uncertainty shocks also propagate through the product term inside the conditional expectation involving the ratio of marginal utilities and next-period labor market tightness. In Figure OB.2, comparing the dashed-dotted line – corresponding to them model with wages determined by Nash bargaining under risk neutrality – and the dotted line – where only the money-metric value of leisure is held constant – reveals that the presence of this term exerts a positive effect on the response of the firm value to a rise in perceived future volatility. The opposite is true for the wage response. To explain why this is the case, note that similar to the precautionary savings effects discussed above, a first consequence of agents anticipating persistently higher future volatility is to put upward pressure on the expected future marginal utility, which in this instance exerts *upward* pressure on the expected future wage. Intuitively, any benefit workers derive from a tight labor market in the future counts for more. This effect lowers expected future dividends and hence the current match value, setting off the by now familiar chain of events that ends up lowering realized labor market tightness and higher unemployment. On the other hand, though, business cycle fluctuations are associated with a negative covariance between the stochastic discount factor and labor market tightness. As

[OB.5] When real wages are assumed to be extrinsically rigid, as in Leduc and Liu (2016), these effects are attenuated.

greater volatility strengthens this negative co-movement, anticipation thereof acts to *lower* the wage in both present and future, which incentivizes job creation. In principle, the net effect of these two channels is ambiguous, but the figure reveals that under the benchmark calibration it, too, serves to dampen the adverse impact of the uncertainty shock on unemployment relative to the case of risk-neutral Nash bargaining.

In summary, when households are risk averse, a rise in uncertainty may push the economy into a recession even when wages, and therefore also dividends, are linear in productivity. Nash bargaining over wages, on the other hand, may exert a dampening effect under risk aversion, provided the workers' total flow benefits from unemployment are sensitive to variations in marginal utility.

Appendix OC Two-period model with heterogeneity

This section presents and examines a two-period variant of the search-and-matching model developed in section 4.1, which features an option-value effect of waiting due to uncertainty about productivity at both the firm-specific and the aggregate levels. This two-period model affords an intuitive, graphical exposition of the key mechanisms as well as analytical results. We first suppose that the probability of an entrepreneur successfully hiring a worker upon posting a vacancy is fixed at some level $\bar{h}$. This case reveals the conditions under which an option value of waiting exists, and the underlying intuition, in a particularly transparent way. Thereafter, we consider the case in which congestion on the matching market renders the hiring rate endogenous, as in the full model.

OC.1 Constant hiring rate

Environment. There are two periods and no discounting. Entrepreneurs can produce either in period 1 or in period 2, but not both.[OC.1] That is, the separation rate is equal to 1. Production is given by $z_t + a_{i,t}$, where a_i is distributed over the interval $[-\bar{a}, +\bar{a}]$ according to the distribution F. Aggregate productivity in the first period is $z_1 = 1$, while z_2 is equal to either $1+\Delta$ ('expansion') or $1-\Delta$ ('recession'), with equal probabilities. The parameter Δ is, thus, a measure of aggregate uncertainty. Workers have no bargaining power and their outside option is zero, such that the profits of a matched entrepreneur are equal to the full value of output net of hiring costs. With the price of such output normalized to unity, and without loss of generality, let the cost of starting a firm, κ, be equal to the fixed hiring probability $\bar{h}$, so that an entrepreneur with draw $a_{i,1} = 0$ in period 1 makes zero profits.

No aggregate uncertainty. Suppose first that there is no aggregate uncertainty, that is, $z_1 = z_2 = 1$. Then the condition pinning down the cutoff productivity of the marginal entrepreneur who

[OC.1] This setup, where there are no opportunities after a production spell, resembles our assumption in the full model that entrepreneurs 'die' – in the sense of having zero value – upon separation.

is indifferent between either entering in the first period or waiting is

$$-\kappa+\bar{h}(1+\widehat{a}_{1,\Delta=0}) = \int_{-\bar{a}}^{\bar{a}} \max\{-\kappa+\bar{h}(1+a),0\}\, dF(a), \tag{OC.1}$$

where $\widehat{a}_{1,\Delta=0}$ denotes the cutoff productivity level in the absence of aggregate uncertainty. Cancelling terms and rewriting the integral, this equation can be written as[OC.2]

$$\begin{aligned}\widehat{a}_{1,\Delta=0} &= \int_{0}^{\bar{a}} af(a)da, \\ &= \underbrace{E[a|a\geq 0]}_{\equiv a_2^*}\underbrace{\text{prob}(a\geq 0)}_{p_2} > 0,\end{aligned} \tag{OC.2}$$

where a_2^* is the expected value of a_2 conditional on being above the period-2 cutoff level, and p_2 denotes the probability of such a draw.

The fact that $\widehat{a}_{1,\Delta=0} > 0$ means that there is a standard option value of waiting due to idiosyncratic uncertainty in this model. Other things equal, the presence of idiosyncratic uncertainty encourages unmatched entrepreneurs to wait, because doing so preserves the optionality of obtaining a better draw in the future and entering; they can always not enter given a poor draw (as indicated by the max-operator), which eliminates downside risk. This mechanism affects the steady state of the main model. [OC.3] Next we will prove that this option value of waiting is amplified if the anticipated variance of z_2, Δ, is positive.

Aggregate uncertainty. The intuitive logic behind the option-value effect due to aggregate uncertainty is that in a time of high overall productivity, the level of idiosyncratic productivity needed to cover vacancy posting costs is lower, so that the probability of having such a draw is higher; at the same time, the expected value of producing conditional upon entry is higher. The opposite is true in times of low productivity, but the decrease in expected profits is lower because

[OC.2]To derive the following expressions, we use two facts: first, an entrepreneur with a draw $a_i < 0$ prefers to not enter in period 2 and, instead, makes zero profits (reflecting the max operator); second, given the fixed hiring rate we can use that $\kappa/\bar{h} = z_1 = z_2 = 1$ to simplify the expression.

[OC.3]One objective of the recalibration procedure described in section 4.2 is, then, to ensure that this option-value effect does not lower vacancy posting (and, hence, raise unemployment) in steady state – "steady state" in the sense of there being no aggregate uncertainty – below the calibration target.

entrepreneurs with low draws avoid entering in the first place and, instead, make zero profits.

To capture this intuition, we can use the same type of expression for the cutoff when there is no aggregate risk. Note first that the cutoff in period 2 satisfies $-\kappa+\bar{h}(z_2+\widehat{a}_2)=0$, so that $\widehat{a}_2=-\Delta$ in good times, and $\widehat{a}_2=+\Delta$ in bad times. Then denoting by $\widehat{a}_{1,\Delta>0}$ the cutoff productivity level in the presence of aggregate uncertainty, the cutoff equation becomes[OC.4]

$$\widehat{a}_{1,\Delta>0}=\frac{1}{2}\int_{-\Delta}^{\bar{a}}(a+\Delta)f(a)da+\frac{1}{2}\int_{+\Delta}^{\bar{a}}(a-\Delta)f(a)da.$$

The different lower limits of integration indicate that when $z_2=1+\Delta$, entrepreneurs with draws above $-\Delta$ will make positive profits, whereas when $z_2=1-\Delta$, only those with draws above $+\Delta$ will do so. Subtracting equation (OC.2) from equation (OC.3), we obtain

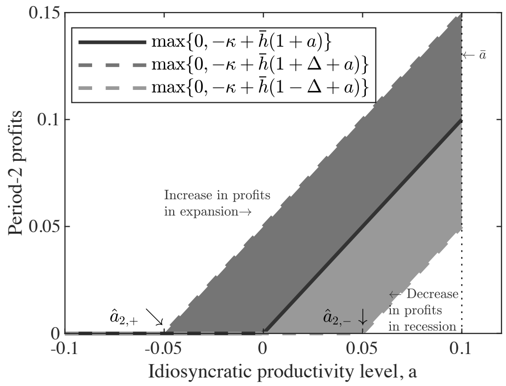
$$\widehat{a}_{1,\Delta>0}-\widehat{a}_{1,\Delta=0}=\frac{1}{2}\left(\int_{-\Delta}^{\bar{a}}(a+\Delta)f(a)da-\int_{0}^{\bar{a}}af(a)da\right)+\frac{1}{2}\left(\int_{+\Delta}^{\bar{a}}(a-\Delta)f(a)da-\int_{0}^{\bar{a}}af(a)da\right).$$

The first term in brackets is positive and the second is negative. However, the first term is larger in absolute value. Figure OC.1a illustrates this point. The dark grey area corresponds to the gain in profits when $z_2=1+\Delta$ rather than $z_2=1$; this is the first term. The lighter grey area describes the loss in profits when $z_2=1-\Delta$ instead; the second term. Clearly, the former area is greater than the latter. The reason is that the entrepreneur has a greater chance of making a draw that yields strictly positive profits in good times. The top-right panel OC.1b illustrates how, when deciding whether to enter or wait in the first period, the greater expected profits in the presence of aggregate risk pushes up the option value of waiting. Consequently, the cutoff level above which entrepreneurs are willing to enter in the first period is higher when $\Delta>0$ than when $\Delta=0$.

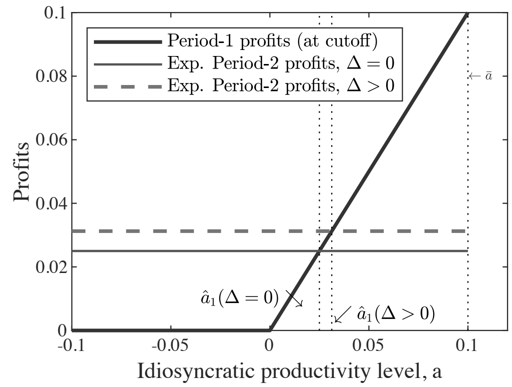
Analytical characterization given uniform distribution. To conclude this section, we analytically characterize the cutoff levels $\widehat{a}_{1,\Delta=0}$ and $\widehat{a}_{1,\Delta>0}$ under a particular functional form assumption

[OC.4] Deriving this expressions involves the same steps noted in Footnote OC.2, adjusted for the fact that the period-2 cutoff now is a function of the stochastic aggregate state, z_2.

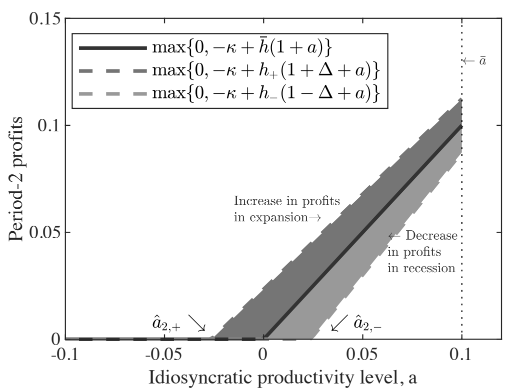
Figure OC.1: Two-period model



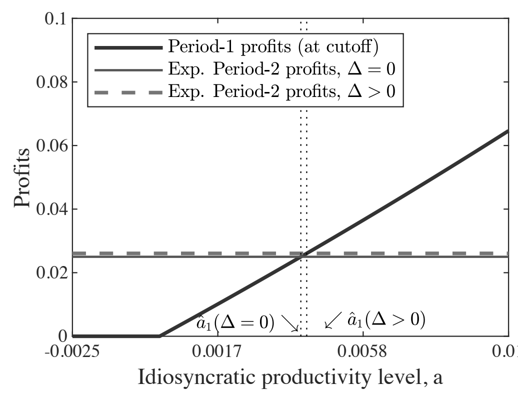
(a) Value of waiting: fixed hiring prob.



(b) Cutoff determination: fixed hiring prob.



(c) Value of waiting: variable hiring prob.



(d) Cutoff determination: variable hiring prob.

Notes: The figure shows how idiosyncratic and aggregate uncertainty both give rise to an option value of waiting in the two-period model. In the upper row, the hiring rate is fixed at $\bar{h}$ (corresponding to $\alpha = 0$), whereas the lower row allows that rate to vary according to a standard matching function ($\alpha = 0.5$). Throughout, idiosyncratic productivity is distributed uniformly over $[-0.1, +0.1]$. To ease visual comparison, the panels with fixed hiring rate assume that $\Delta = 0.05$, whereas in the case of a variable hiring rate, $\Delta = 0.15$. Finally, notice that in Panel OC.1d, period-1 profits are computed as the left-hand side of equation (OC.6), so that the slope incorporates endogenous changes in the hiring rate. To facilitate a straightforward comparison of alternative vacancy posting cost interpretations in section OF.1, the steady-state hiring rate is set equal to one. As a consequence, $h_- > 1$, which is, strictly speaking, inconsistent with h describing a probability, but intuitively just means that one entrepreneur could employ more than one worker.

about the distribution F, namely, that a_i is uniformly distributed over $[-\bar{a}, +\bar{a}]$. Then

$$p_t = \text{prob}(a \geq \widehat{a}_t) = \frac{\bar{a} - \widehat{a}_t}{2\bar{a}}, \tag{OC.3}$$

$$a_t^* = E[a | a \geq \widehat{a}_t] = \frac{\bar{a} + \widehat{a}_t}{2}. \tag{OC.4}$$

To make the connection between the two-period model and the full model very explicit, write the cutoff condition as follows:

$$-\kappa + \bar{h} \underbrace{(1 + \widehat{a}_1)}_{J_1(\widehat{a}_1)} = 0 + E_1 \left[p_2(-\kappa + \bar{h} \underbrace{(z_2 + a_2^*)}_{J_2(a_2^*)}) + (1 - p_2)0 \right].$$

The expectations operator E_1 conditions on the information available in period $t = 1$. The entire right-hand side describes the value of waiting, J^U. It proves instructive to re-write it as follows:

$$\begin{aligned} J^U &= -\kappa E_1[p_2] + \bar{h} E_1[p_2 J_2(a_2^*)] \\ &= -\kappa E_1[p_2] + \bar{h} \Big(E_1[p_2] E_1[J_2(a_2^*)] + \text{Cov}_1[p_2, J_2(a_2^*)] \Big). \end{aligned}$$

To characterize the expectation terms on the right-hand side, use the fact that $\widehat{a}_2 = -\Delta$ in good times, and $\widehat{a}_2 = +\Delta$ in bad times. Thus,

$$\begin{aligned} E_1[p_2] &= \frac{1}{2}\left(\frac{\bar{a} - \Delta}{2\bar{a}}\right) + \frac{1}{2}\left(\frac{\bar{a} + \Delta}{2\bar{a}}\right) = \frac{1}{2}, \\ E_1[J_2(a_2^*)] &= \frac{1}{2}\left(1 + \Delta + \frac{\bar{a} - \Delta}{2}\right) + \frac{1}{2}\left(1 - \Delta + \frac{\bar{a} + \Delta}{2}\right) = 1 + \frac{\bar{a}}{2}, \\ \text{Cov}_1[p_2, J_2(a_2^*)] &= E_1\{(p_2 - E_1[p_2])(J(a_2^*) - E_1[J(a_2^*)])\} = \frac{\Delta^2}{4\bar{a}} \geq 0. \end{aligned}$$

Substituting these expressions into equation (OC.4) and cancelling terms, we find that the cutoff-level of productivity in period $t = 1$ above which entrepreneurs enter is

$$\widehat{a}_1 = \frac{\bar{a}}{4} + \frac{\Delta^2}{4\bar{a}}. \tag{OC.5}$$

The cutoff level is positive even in the absence of aggregate uncertainty, $\widehat{a}_{1,\Delta=0} = \frac{\bar{a}}{4}$, but the option value of waiting is greater when Δ is positive due to the covariance term, with $\frac{\partial \widehat{a}_1}{\partial \Delta} = \frac{\Delta}{2\bar{a}}$. Intuitively, what this covariance term captures is the following. When the aggregate state of the economy is high, even entrepreneurs with relatively low idiosyncratic productivity can recover the costs of posting a vacancy through production, so that the probability of having a draw that is sufficiently good to incentivize entry is greater. At the same time, the expected value of producing conditional upon entry is higher. The opposite is true in times of low productivity – when only few entrepreneurs draw sufficiently high idiosyncratic productivity values – but the decrease in expected profits is lower, because entrepreneurs with low draws avoid entering in the first place and, instead, make zero profits. An increase in aggregate volatility makes the covariance term correspondingly larger. To illustrate, Figures OC.1a and OC.1b are drawn for $\bar{a} = 0.1$ and $\Delta = 0.05$, such that $\widehat{a}_{1,\Delta=0} = 0.025$ and $\widehat{a}_{1,\Delta=0} = 0.03125$.

It is interesting to note that the effect on the cutoff from a change in the period-2 variance of aggregate productivity is larger for smaller values of $\bar{a}$ and becomes ill-defined as $\bar{a}$ approaches zero. The intuitive explanation is that when idiosyncratic dispersion is small, firms are clustered around the cutoff.[OC.5] In particular, when $\bar{a}$ is close to zero, tiny changes in Δ would imply hitting the bounds of the distribution. This is not taken into account by the derivative. For this reason, it is instructive to consider the case without idiosyncratic risk.

Aggregate uncertainty but no idiosyncratic uncertainty. Suppose that $a_{i,1} = a_{i,2} = 0$ for all firms, and that there is a unit mass of firms. Then in the absence of aggregate uncertainty, all entrepreneurs are indifferent between creating a firm and not doing so. Now suppose that a fraction $\frac{1}{2}$ of all firms does create a firm in the first period. Even a tiny introduction of aggregate uncertainty regarding z_2 will induce a discontinuous jump from $\frac{1}{2}$ to 1. This is effectively what the derivative $\frac{\partial \widehat{a}_1}{\partial \Delta} = \frac{\Delta}{2\bar{a}}$ tells us.

[OC.5] A second objective of our recalibration procedure is to counteract the fact that for greater degrees of dispersion changes in the cutoff level induce smaller changes in the probability of entry and, hence, the number of jobs created. To this end, we adjust the sensitivity of entrepreneurial profits to changes in aggregate productivity upward when idiosyncratic dispersion is greater. (This is achieved by making the entrepreneurial profit share in output smaller, an aspect we abstract from in the two-period model.)

OC.2 Variable hiring rate

Instead of the hiring rate being invariant to economic conditions, suppose now that there is congestion in the labor market.

Environment. Matches are determined according to a Cobb-Douglas function, as in the full model. As the separation rate is one and 'dead' entrepreneurs are replaced by new ones, the number of vacancies posted is simply $v_t = p_t \Upsilon$, so that the hiring rate is $h_t = \psi(\Upsilon p_t)^{-\alpha}$, where Υ is the mass of entrepreneurs, ψ is the matching efficiency, and $(1-\alpha)$ the elasticity of matches with respect to vacancies. As the goal here is analytical clarity rather than targeting a particular unemployment rate, we set Υ equal to one. To ensure that in the absence of uncertainty we have that $h = \bar{h}$ and $p = 1/2 \Leftrightarrow \widehat{a} = 0$, set $\psi = \bar{h}\left(\frac{1}{2}\right)^{\alpha}$ and $\kappa/\bar{h} = 1$.[OC.6]

Graphical analysis. Three key implications of the amended description of the model environment can be understood immediately from the graphical exposition in Figure OC.1. In particular, the bottom row of panels parallels the top row in terms of their construction, but now the hiring rate is endogenous.

Firstly, as before, there exist option-value effects due to idiosyncratic uncertainty ($\widehat{a}_{1,\Delta=0} > 0$) and due to aggregate uncertainty ($\widehat{a}_{1,\Delta>0} > \widehat{a}_{1,\Delta=0}$). In particular, the figure underscores that greater anticipated uncertainty over period-2 aggregate productivity renders expected period-2 profits higher, strengthening any motive for entrepreneurs to wait due to idiosyncratic productivity dispersion. As such, endogenous matching does not alter the central, qualitative message of section OC.1. There are, however, two forces that dampen the magnitude of such option-value effects.

For one, the sensitivity of the value of waiting to aggregate uncertainty is lessened. As Panel OC.1c shows, both the *increase* in profits in an expansion and the *decrease* in profits in a recession are muted relative to the case of a fixed hiring rate. Intuitively, in an expansion, congestion on the entrepreneurs' side of the matching market means that the hiring rate is lower than in steady state or when the hiring rate is fixed (as reflected in the flatter slope of the dashed profit line). Hence, the increase in profits is muted, as more vacancies posted remain unfilled. The opposite is true in a

[OC.6] In a model with positive wages, we would set $\kappa/\bar{h} = Q$, where Q denotes steady-state profits.

recession, when the probability of a successful match conditional on posting a vacancy is greater. Because the lower hiring rate in an expansion applies to a wider range of idiosyncratic productivity values, the net effect is to limit the rise in expected profits due to aggregate uncertainty. Therefore, endogenous matching dampens the rise in the option value of waiting when uncertainty over future aggregate productivity is elevated.

More importantly still, for any given change in the value of waiting, the increase in the period-1 cutoff and, thus, the impact of period-2 uncertainty on period-1 economic activity, is much smaller than when the hiring rate is fixed. For any given increase in expected period-2 profits due to either idiosyncratic or aggregate uncertainty, the equilibrium change in the period-1 cutoff and, hence, vacancy posting, is much smaller (Panel OC.1d). The reason is that any increase in the cutoff lowers the share of entrepreneurs entering and, hence, the amount of congestion. The endogenous rise in the hiring rate makes entering more attractive, leading to a partial offsetting of the increase in the cutoff that pushed up the hiring rate in the first place. The converse holds for a fall in the cutoff. This general equilibrium dampening force is suppressed when the hiring rate is constant.

While the graphical analysis communicates the central implications of allowing for matching frictions, for the interested reader we next consider again in detail the different configurations of idiosyncratic and aggregate uncertainty. We start with the case where idiosyncratic dispersion is zero but aggregate uncertainty is positive, as in the benchmark search-and-matching model.

Aggregate uncertainty but no idiosyncratic uncertainty. Suppose first that $a_{i,1} = a_{i,2} = 0$ for all firms. As in the case of a fixed hiring rate, in the absence of aggregate uncertainty all entrepreneurs are indifferent between creating a firm and not doing so. Suppose that initially a fraction $\frac{1}{2}$ does. In a marked difference from the previous case, now the introduction of a tiny amount of aggregate uncertainty does *not* lead to a discontinuous jump from $\frac{1}{2}$ to 1. To the contrary, when Δ is positive (but small), the entry probability p_2 and, hence, the hiring probability h_2 adjusts such that expected profits are equal to zero both when $z_2 = 1+\Delta$ and when $z_2 = 1-\Delta$.[OC.7] As such,

[OC.7] As emphasized in section 4, this argument presumes that the constraint on the available number of entrepreneurs is sufficiently slack – respectively, the shocks sufficiently small –so that movements in the hiring rate can render expected profits equal to zero.

there is no option value of waiting and p_1 is still equal to $1/2$. Thus, the endogenous adjustment of the hiring probability plays a key role in ensuring that the free-entry condition holds in every state of the world.

Idiosyncratic uncertainty but no aggregate uncertainty. Next, consider the option value of waiting due to firm-level dispersion but absent aggregate uncertainty. With a variable hiring rate, the cutoff equation becomes

$$-\kappa+\underbrace{\psi p_1^{-\alpha}}_{h_1}(1+\widehat{a}_1)=p_2\left(-\kappa+\underbrace{\psi p_2^{-\alpha}}_{h_2}(1+\widehat{a}_2)\right)>0,$$

and it implies that the value of waiting in period 1 still is positive. For notice that our calibration ensures that when $\Delta=0$, the period-2 cutoff is always equal zero, with half of all entrepreneurs entering, so that $h_2=\bar{h}$. But the conditional expected value is positive provided $\bar{a}>0$, that is, $a_2^*>\widehat{a}_2=0$.[OC.8] Thus, an increase in idiosyncratic dispersion, that is in $\bar{a}$, unambiguously raises the right-hand side by raising a_2^*. For the left-hand side to increase also, it must be that $\widehat{a}_1$ increases, as p_1 is strictly decreasing in $\bar{a}$ when $\widehat{a}_1>0$.

In contrast to the case with fixed matching probabilities, however, $\widehat{a}_1$ increases more gradually as the degree of idiosyncratic dispersion increases. The reason is that a variable hiring rate introduces a natural dampening effect: when the cutoff is higher and more entrepreneurs choose to wait, the hiring probability increases, which makes entering more attractive again. Specifically, whereas in the case of a fixed hiring rate, $\frac{\partial J_1(\widehat{a}_1)}{\partial \widehat{a}_1}=\bar{h}$, we now have

$$\frac{\partial J_1(\widehat{a}_1)}{\partial \widehat{a}_1}=h_1+(1+\widehat{a}_1)\frac{\partial h_1}{\partial \widehat{a}_1}, \qquad \text{where}$$
$$\frac{\partial h_1}{\partial \widehat{a}_1}=\frac{\alpha}{2\bar{a}}\left(\frac{\bar{a}-\widehat{a}_1}{2\bar{a}}\right)^{-(\alpha+1)}.$$

As such, a small increase in $\widehat{a}_1$ following a rise in idiosyncratic uncertainty is sufficient for the

[OC.8] The period-2 cutoff level, $\widehat{a}_2$, satisfies $-\kappa+\psi p_2^{-\alpha}(1+\widehat{a}_2)=0$ and given the uniform distribution we have that $p_2=\frac{\bar{a}-\widehat{a}_2}{2\bar{a}}$.

left-hand side to be equal to the – now greater – right-hand side.[OC.9]

Idiosyncratic and aggregate uncertainty. Finally, when both $\bar{a} > 0$ and $\Delta > 0$, the cutoff equation with endogenous matching is

$$-\kappa + h_1(1+\widehat{a}_1) = E_1\left[p_2(-\kappa + h_2(z_2 + a_2^*))\right]. \tag{OC.6}$$

As in the case without aggregate uncertainty that we just considered, for any given change in $\widehat{a}_1$, the fact that h_1 is an increasing function of $\widehat{a}_1$ introduces a first dampening force relative to the model with fixed hiring rate. What about the magnitude of the value of waiting?

Consider the right-hand side of equation (OC.6), that is, J^U. In period 2, we have the following two expressions that implicitly define the cutoff levels (recall that p_2 is a function of the cutoff itself).

$$\begin{aligned}
-\kappa + h_{2,+}(1+\Delta+\widehat{a}_{2,+}) = 0 \Leftrightarrow \widehat{a}_{2,+} &= \frac{\kappa}{\psi}\left(\frac{\bar{a}-\widehat{a}_{2,+}}{2\bar{a}}\right)^{\alpha} - 1 - \Delta, \\
-\kappa + h_{2,-}(1-\Delta+\widehat{a}_{2,+}) = 0 \Leftrightarrow \widehat{a}_{2,-} &= \frac{\kappa}{\psi}\left(\frac{\bar{a}-\widehat{a}_{2,-}}{2\bar{a}}\right)^{\alpha} - 1 + \Delta.
\end{aligned}$$

Thus, relative to the case with fixed hiring rate, the cutoff in an expansion, $\widehat{a}_{2,+}$, is higher, while the cutoff in a recession, $\widehat{a}_{2,-}$, is lower. The reason is that the effective hiring costs to be covered by revenue over the duration of a match are greater (smaller) in an expansion (recession), when the hiring probability is lower (higher).

[OC.9] This relationship is non-linear, insofar as a change in the cutoff of a given size leads to a more modest change in the entry probability and, hence, hiring rate for a more dispersed distribution.

We can rewrite the value of waiting as a a function of these cutoffs.

$$
\begin{aligned}
J^U &= \frac{1}{2} E\left[p_{2,+}\left(-\kappa+\psi(p_{2,+})^{-\alpha}(1+\Delta+a)\right)|a \geq \widehat{a}_{2,+}\right] \\
&+ \frac{1}{2} E\left[p_{2,-}\left(-\kappa+\psi(p_{2,-})^{-\alpha}(1-\Delta+a)\right)|a \geq \widehat{a}_{2,-}\right], \\
&= \frac{1}{2}\left[-\kappa\left(\frac{\bar{a}-\widehat{a}_{2,+}}{2\bar{a}}\right)+\left(\psi\left(\frac{\bar{a}-\widehat{a}_{2,+}}{2\bar{a}}\right)^{1-\alpha}\left(1+\Delta+\frac{\bar{a}+\widehat{a}_{2,+}}{2}\right)\right)\right] \\
&+ \frac{1}{2}\left[-\kappa\left(\frac{\bar{a}-\widehat{a}_{2,-}}{2\bar{a}}\right)+\left(\psi\left(\frac{\bar{a}-\widehat{a}_{2,-}}{2\bar{a}}\right)^{1-\alpha}\left(1-\Delta+\frac{\bar{a}+\widehat{a}_{2,-}}{2}\right)\right)\right].
\end{aligned}
$$

While no explicit solution is possible, the expressions nevertheless reveal very intuitive implications of making the hiring rate endogenous.[OC.10] Recall that in the case of a fixed hiring rate (nested here for $\alpha = 0$), the option value of waiting was increasing in the anticipated variance of period-2 productivity, Δ, because the entry probability and the conditional expected value of a match co-varied positively, that is, $\text{Cov}_1[p_2, J_2(a_2^*)] > 0$. This same effect is still operative here, but its quantitative magnitude is dampened, because even though in an expansion the probability of entry is higher, congestion externalities mean that the hiring probability is lower, which limits the conditional expected value of entering. The converse holds true in a recession. That is, $\text{Cov}_1[h_2, J_2(a_2^*)] < 0$. The expressions also underscore that since the hiring rate only ever moves due to changes in the entry probability, this mechanism can only ever dampen but not completely eliminate the option value of waiting due to aggregate uncertainty, provided that $\bar{a} > 0$.

In summary, the two-period model illustrates the existence of option-value considerations in a model with a finite mass of entrepreneurs and firm-specific productivity. Allowing for congestion in the matching market is crucial because it ensures that introducing even the tiniest amount of aggregate uncertainty does not lead to a discontinuous rise in the value of waiting. At the other end of the extreme, in the absence of heterogeneity in firm-specific productivity (and presuming a sufficiently large number of potential entrepreneurs), such endogenous variations in the hiring probability entirely eliminate any option-value effects. The model with a variable hiring rate, as in

[OC.10] An additional mechanism is that the rise in the hiring rate in a recession is greater than its decline in a boom – the matching function being concave – so that aggregate uncertainty raises the expected hiring rate.

the standard search-and-matching environment, and a finite mass of entrepreneurs that vary in terms of their productivity draws represents a potentially attractive middle way.

Appendix OD Variations on the heterogeneous-firm model

The heterogenous-firm version of the search-and-matching (SaM) model developed and analyzed in the main text supposes that entrepreneurs “die” upon separation; and that the mass of of potential entrepreneurs is both finite and constant. This section shows how the model may be modified or extended to allow for a relaxation of these two assumptions, respectively, and with what implications. These examinations shed further light on, and underscore the importance of, the mutual exclusivity property that needs to obtain in order for an option-value channel of waiting to exist through which uncertainty shocks may affect macroeconomic activity.

OD.1 Infinitely-lived entrepreneurs

In the baseline model, entrepreneurs have finite lives; specifically, following a separation shock, the entrepreneur “dies” and gets replaced. Accordingly, the match value given idiosyncratic productivity a is

$$J_t(a) = (1-\omega)(\bar{x}(z_t+a)-\chi) + \beta(1-\delta)E_t[J_{t+1}(a)]. \tag{OD.1}$$

This specification has the benefit of rendering the transmission of uncertainty shocks very transparent, since holding constant a and the sequence of aggregate productivity levels, $\{z_t\}_{t=0}^{\infty}$, the match value, $J_t(a)$, is invariant to an increase in perceived volatility. Consequently, any macroeconomic effects triggered by such an increases can be traced back solely to variations in the (expected) value of waiting, $E_t[J^U_{t+1}]$, consistent with the standard option-value mechanism for postponing investment. Lastly, it ought to be noted that equation (OD.1) closely mimics the firm value in the baseline model with homogenous firms.

An alternative model supposes, instead, that entrepreneurs are infinitely lived. Specifically, suppose that following separation the entrepreneur “survives” and with probability $(1-\gamma)$ she can immediately draw a new idiosyncratic value, a, but that she remains idle for one period with the complementary probability (and in the next period this process repeats itself). Thus, $1-\gamma$ is the

Poisson arrival rate of new "business projects", and $1/(1-\gamma)$ is the expected duration of "idleness" following separation.

In this setup, the value of a matched entrepreneur given idiosyncratic productivity a is as follows:[OD.1]

$$J_t(a) = (1-\omega)(\bar{x}(z_t + a) - \chi) + \beta(1-\delta)E_t[J_{t+1}(a)] + \beta\delta E_t[\hat{J}^U_{t+1}], \tag{OD.2}$$

where

$$\hat{J}^U_t = (1-\gamma)J^U_t + \beta\gamma E_t[\hat{J}^U_{t+1}]. \tag{OD.3}$$

denotes the value of a newly separated, idle entrepreneur. Here, J^U_t continues to be the value of an unmatched entrepreneur prior to the revelation of her a in period t.

Moreover, the equation giving the number of vacancies posted needs to be adjusted to account for the fact that some of the unmatched entrepreneurs are unavailable because they have not yet had the chance to draw a new productivity value. Equation (21) is, thus, replaced by

$$v_t = p_t\left(\Upsilon - (1-\delta)n_{t-1} - \delta\gamma s_{t-1}\right), \tag{OD.4}$$

$$s_t = n_t + \gamma s_{t-1}. \tag{OD.5}$$

All remaining equations are unchanged. In particular, the cutoff-level of idiosyncratic productivity continues to be pinned down by

$$h_t(J_t(\hat{a}_t) - \beta E_t[J^U_{t+1}]) - \kappa = 0. \tag{OD.6}$$

To identify how this model variation affects the transmission of uncertainty shocks relative to

[OD.1] See section OE.2 of the online appendix for a derivation. There we show that this model may be interpreted as describing the vacancy-posting decisions of a representative firm with multiple business projects, some of which are matched with a work and, hence, productive, while others are idle.

the baseline, both qualitatively and quantitatively, it is instructive to initially consider the special case where γ is set to zero. This means that upon separation any entrepreneur simply switches to the state of being an unmatched entrepreneur who, in each subsequent period in which she remains unmatched, can draw a new a. In that case, relative to the baseline model with finitely-lived entrepreneurs the only equation that is changed is that for the match value. Namely, for $\gamma = 0$, equation (OD.2) simplifies to

$$J_t(a) = (1-\omega)(\bar{x}(z_t + a) - \chi) + \beta E_t \left[(1-\delta) J_{t+1}(a) + \delta J^U_{t+1} \right]. \tag{OD.7}$$

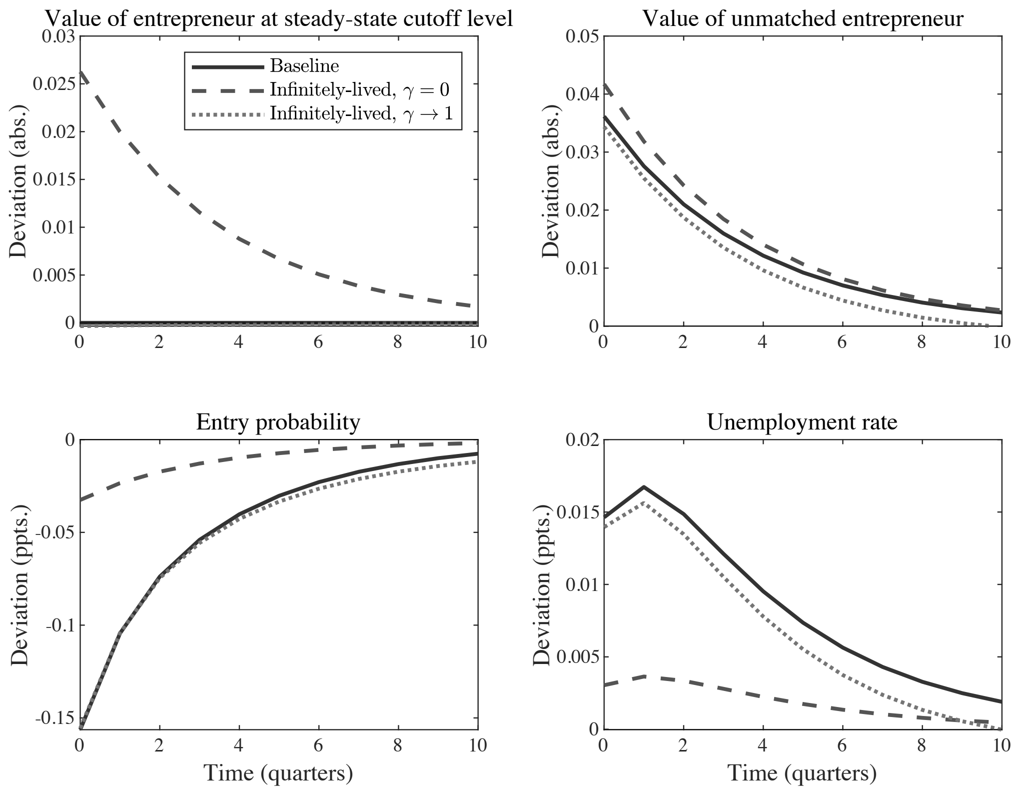
According to this specification, an entrepreneur that commits and creates a job in the current period still benefits from increased uncertainty, although only at some point in the future, namely after the created relationship ends. The option-value of waiting effect considered in the main text is still present in this version, however, its effect is dampened since increased uncertainty now affects both the option of waiting and the commitment to investment immediately.

Figure OD.1 provides a quantitative examination, whereby the solid lines indicate pure uncertainty effects in the baseline model with finitely lived entrepreneurs and the dashed version corresponds to the alternative version with infinitely-lived entrepreneurs and γ set to zero.[OD.2] This numerical analysis features certain intricacies that warrant explanation. For one thing, preserving comparability requires that the two models are individually calibrated according to the procedure described in 4.2, that is, taking into account the adjusted form of the match value equation (OD.7). Else, if we imposed the same parameter values on both models, in one case we would miss the calibration targets. Specifically, notice that for a given set of parameter values and σ_a, vacancy-posting is generally, and other things equal, more attractive relative to staying idle in the model with infinitely lived entrepreneurs.[OD.3] As such, for a given value of σ_a, the model without entrepreneur death and $\gamma = 0$ is closer to the homogeneous-firm environment than the baseline. As a consequence,

[OD.2] To aid orientation, observe that the solid lines in Figure OD.1 match the solid lines in Figure 5.

[OD.3] For instance, given the parameter values used for the homogeneous-firm model, the steady-state unemployment rate in the heterogeneous-firm model with infinitely-lived entrepreneurs and $\gamma = 0$, given $\sigma_a = 0.003$, is 6.58 percent (versus a targeted value of 6.4 percent). For comparison, the equivalent value in the model with entrepreneur death is 9.71 percent (see Figure 4).

Figure OD.1: Entrepreneur death (baseline) vs. infinitely-lived entrepreneurs



Notes: The figure reports pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$. In the models corresponding to the solid and dashed lines, the model parameters are recalibrated following the procedure described in section 4.2 and using the respectively applicable model equations, i.e., equation (OD.1) for the baseline model with death ($\sigma_a = 0.003$) and equation (OD.7) for the model with infinitely-lived entrepreneurs and $\gamma = 0$ ($\sigma_a = 0.025$). For $\gamma \to 1$ (approximated by setting $\gamma = 0.999$) we impose the same parameters as in the baseline setting.

the upper bound on σ_a in the recalibrated, alternative model is greater than in the model with death, that is, around 0.025 instead of 0.003. To maximize comparability, the IRFs for each model are therefore computed by imposing these respective (approximate) maximum values for σ_a.

Turning to the analysis, the respective pure uncertainty effects on the value functions are intuitive. Whereas in the baseline model, the value of a matched entrepreneur for a given level of idiosyncratic productivity – in this instance, the steady-state cutoff level – does not respond to an increase in uncertainty (solid line), in the alternative specification with $\gamma = 0$ the value of entrepreneurs who are already matched increases as well (dashed line). It is because of this difference that, even though the rise in the value of remaining unmatched is roughly similar in both specifications, the ultimate effects on entrepreneurial entry probabilities and worker unemployment are dampened in the model with infinitely-lived entrepreneurs relative to the baseline. Indeed, the recalibrated model with infinitely-lived entrepreneurs and $\gamma = 0$ implies an increase in unemployment following an uncertainty shock that is similar in magnitude to the effect obtained in the homogeneous-firm model with Nash bargaining (cf. Figure 1), which is substantially smaller than the baseline.

It is tempting to infer from this two-way comparison that what matters for the magnitude of the option-value effect is whether entrepreneurs are finitely or infinitely lived. That is not quite correct. To see this, consider now the model with infinitely-lived entrepreneurs but setting γ greater than zero. In particular, let $\gamma \to 1$ and adjust the measure of potential entrepreneurs to reflect that a fraction of unmatched entrepreneurs that is increasing in γ are not in a position to post vacancies. With $\Upsilon = \Upsilon^{\text{baseline}} + n\delta \frac{\gamma}{1-\gamma}$, in the limit the steady states of the models with finitely-lived and infinitely-lived entrepreneurs coincide. Moreover, turning to the dynamic effects of uncertainty shocks, the dotted line in Figure OD.1, which is based on the model with infinitely-lived entrepreneurs and $\gamma = 0.999$, almost coincides with the baseline model.[OD.4] This third point of comparison thus underlines that the key question concerns the extent to which uncertainty differentially raises the expected value of waiting as opposed to committing to investing in the present. Even when entrepreneurs are infinitely lived, if committing to a hire now and investing in a different project in the future are mutually exclusive – with certainty in the baseline setup and stochastically in the model presented here – then an increase in aggregate uncertainty makes waiting relatively more attractive, with adverse consequences for hiring activity.

We conclude this section by noting that the model with infinitely-lived entrepreneurs considered in this section still remains unrealistic in that, even while the entrepreneur lives on following a separation, the entrepreneur's firm (or business project) does not. That is, the SaM structure implies that expected firm life coincides with the time we expect the worker to stay with one firm, which is a short span of time (the expected firm life implied by our calibration is two and a half years whereas the uncertainty shock is quite persistent). Clearly, in reality firms are not destroyed and their idiosyncratic productivity independently re-drawn – sooner or later, on average, depending on the

[OD.4] The reason why the two models do not entirely coincide is that in the model with infinitely-lived entrepreneurs, those have to (stochastically) wait for projects to be available again. Hence, past separations affect the likelihood of finding new workers, as is reflected in the state variable s_t.

value of γ, when a worker leaves a new one is hired. An appropriate description of empirical reality suitable for quantitative analysis ought to instead feature idiosyncratic firm productivity that is highly persistent but not permanent. Even under a parameterization like $\gamma = 0$, such a model would imply less dampening relative to the baseline model. However, analyzing such a model requires keeping track of the full distribution of existing firm productivities and is beyond the scope of the current paper. What this section has demonstrated, leveraging the transparent cutoff formulation our framework affords, is that an option-value of waiting effect can propagate uncertainty shocks irrespective of whether entrepreneurs have finite or infinite lives.

OD.2 Endogenous measure of entrepreneurs

The baseline version of the heterogeneous-firm model supposes that the mass of potential entrepreneurs, denoted by Υ, is finite and constant. This stands in sharp contrast with the standard SaM framework, in which there is a potentially infinite mass of entrepreneurs. In this appendix, we discuss an extension of our model which endogenizes the mass of potential entrepreneurs, Υ, who, when idle, receive an idiosyncratic shock, a, enter the matching market when a is sufficiently high, and – if matching is successful – start producing.[OD.5] This model extension further clarifies the conditions under which an option-value of waiting mechanism is operative in SaM environments.

Suppose, then, that at the very beginning of a given period, any one of a potentially infinite number of homogeneous individuals can pay a fixed cost μ to obtain the chance to become such an entrepreneur. As in the standard SaM framework, the outside option for these individuals is equal to zero. Upon paying μ this individual becomes an (initially unmatched) entrepreneur with a time-varying probability $g(\Upsilon_t)$, where g is weakly decreasing in Υ_t. That is, if the number of entrepreneurs is already large, then it is less likely that the new-joiner finds a potential business opportunity, or project, that could be matched to a worker.[OD.6]

[OD.5] We thank an anonymous referee for encouraging us to explicitly discuss an extension of this type.

[OD.6] Observe that g is decreasing in the *stock* of already existing projects rather than the measure of potential entrepreneurs looking to enter. In that respect, the formulation here differs from the use of a standard, matching function involving vacancies and unemployed job seekers. Intuitively, the latter centers on congestion externalities that each side imposes on competing members of their own side. By contrast, here a new-joiner's chances are not only negatively

Accordingly, for $\sigma_a > 0$ the now potentially time-varying total mass of entrepreneurs, Υ_t, satisifies the free-entry condition

$$\mu = g(\Upsilon_t) J_t^U, \tag{OD.8}$$

where J_t^U is the value of an idle entrepreneur (prior to the revelation of this period's idiosyncratic draw) given by equation (20).

It warrants emphasizing that equation (OD.8) is meaningful only because in our modified SaM model with cross-sectional productivity dispersion, and in marked contrast to the standard setup, the value of an unmatched entrepreneur, J_t^U, is no longer zero by construction. It is precisely because being an idle entrepreneur carries an option value that it may be worth paying the fixed cost μ. For that idle entrepreneur has some probability of receiving an idiosyncratic productivity, in the present period or in the future, which is such that the expected profits of posting a vacancy, net of the cost κ of doing so, are strictly positive. In the standard model, on the other hand, there is no value to entering as an entrepreneur.

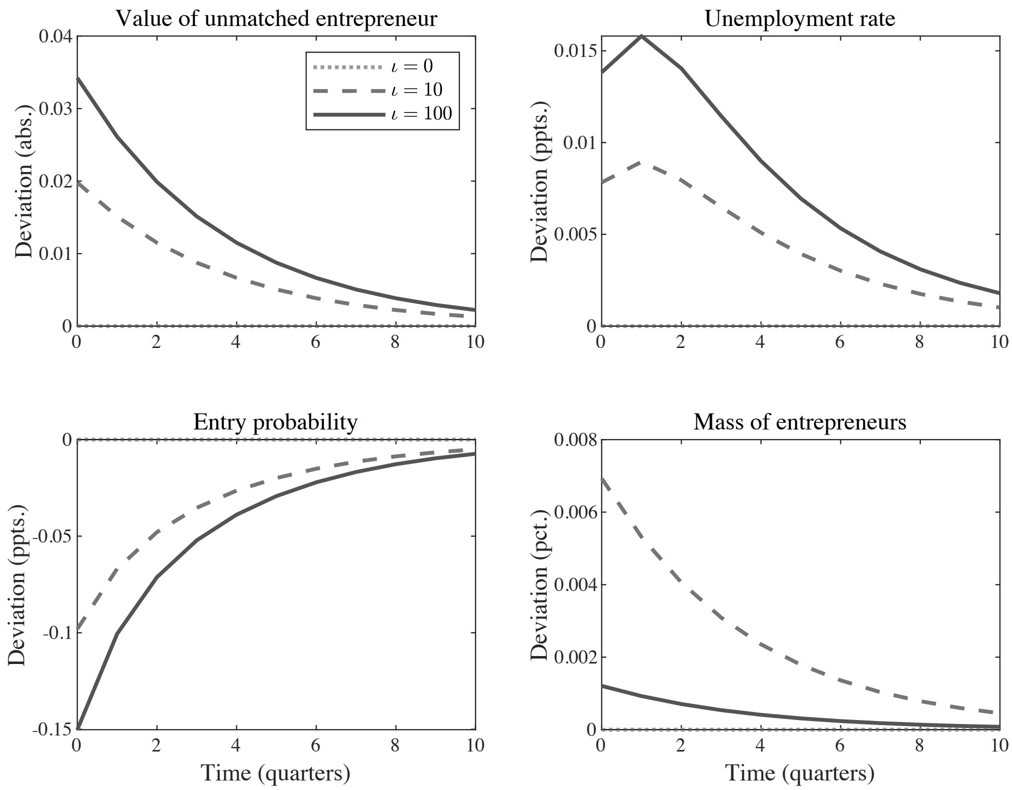
We take g to have a simple, isoelastic form with constant elasticity $\iota \geq 0$, i.e., $g(\Upsilon) = \Upsilon_t^{-\iota}$. Then the mass of entrepreneurs is simply $\Upsilon_t = \left(\frac{J_t^U}{\mu}\right)^{\frac{1}{\iota}}$, with $\frac{1}{\iota}$ measuring the elasticity of entrepreneurial entry with respect to the (scaled) value of being an unmatched entrepreneur.[OD.7] All remaining parameters are unchanged from the baseline calibration described in the main text.

Figure OD.2 indicates the pure uncertainty effects on selected variables of a volatility shock in this economy, conditional upon different entry elasticities. Consider the solid line first. When ι is very large, we are effectively in the world of our modified SaM model with a fixed mass of entrepreneurs as described in Section 4. An increase in aggregate uncertainty pushes up the value of waiting; the mass of entrepreneurs, Υ_t, remains almost constant because even a tiny increase in Υ_t

affected by other new-joiners but also by established entrepreneurs. The idea is that if established entrepreneurs have already picked the low-hanging fruits, this makes it harder for others to come up with viable projects.

[OD.7]This functional form assumption means that g cannot strictly speaking be interpreted as a probability, as the range of g is not restricted to the interval $[0,1]$. So a better interpretation for g is the number or fraction of projects created when the individual pays μ, which could be a fraction of a project when $g < 1$ and more than one project when $g > 1$. Moreover, this choice of functional form maximizes the comparability to the standard SaM model described in Section 2, where the hiring probability h takes an analogous form.

Figure OD.2: Endogenous measure of potential entrepreneurs



Notes: The figure reports pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$. The model parameters are recalibrated following the procedure described in section 4.2.

sharply raises the cost of entrance; and idle entrepreneurs require a higher value for a to warrant vacancy posting due to the increase in J_t^U. Consequently, unemployment rises.

At the other extreme, when $\iota = 0$, the pure uncertainty effects on the unemployment rate are uniformly zero. They thus coincide with the results documented for the standard SaM model in Section 3 given a linear wage rule – despite the fact that there is dispersion in idiosyncratic productivity. When $\iota = 0$, g_t is constant which in turn implies that J_t^U is constant. And although J_t^U is positive rather than zero as in the standard SaM model, it being constant implies that there cannot be an increase in the option value of waiting when uncertainty increases. As shown in the figure, there is no change in the unemployment rate. Thus, just as a perfectly elastic entry-margin at the level of vacancy-posting eliminated any pure uncertainty effects in a standard SaM model, so does perfectly elastic entry at the level of entrepreneurs in this modified model with heterogeneous firms when $\iota = 0$. Lastly, the dashed line in Figure OD.2 captures the intermediate case where an increase in the number of entrepreneurs following a rise in uncertainty dampens, without eliminating, the adverse consequences of uncertainty for vacancy posting and, ultimately, for unemployment.

Appendix OE Representative-firm formulations

The main analysis adopts the common approach used in the search-and-matching (SaM) literature in which there are n_t basic production units and each consists of an entrepreneur who has successfully hired a single worker by investing in a vacancy. We refer to this as the "baseline" formulation in the following. It is well known that this baseline formulation is equivalent to a representative-firm formulation with n_t workers for the standard SaM model (see, e.g., Petrosky-Nadeau *et al.* (2018)). This appendix shows that there also exists a representative-firm formulation with multiple workers for our proposed alternative SaM model with heterogeneity in idiosyncratic productivity. Thus, the framework is silent with regards to if movements in the unemployment rate are driven by existing firms changing their stock of employees, or if there is variations in the creation of new firms. Both interpretations are equally consistent within the framework.

OE.1 Standard model

We first briefly remind the reader of the equivalence between the two environments for the standard SaM model. Recursively written, the Bellman equation for a firm with n_{t-1} employees is

$$v(n_{t-1}, z_t) = \max_{n_t, v_t} \{\bar{x} z_t n_t - w_t n_t - \kappa v_t + \beta(1-\delta) E_t V(n_t, z_{t+1})\}. \tag{OE.1}$$

and its perceived law of motion reads

$$n_t = (1-\delta) n_{t-1} + h(\theta_t) v_t. \tag{OE.2}$$

The firm takes as given the wage w_t and the hiring probability $h(\theta_t)$.

Denoting by J_t the Lagrange multiplier on equation (OE.2), the first-order condition for vacancies then reads

$$0 = -\kappa + h(\theta_t)J_t, \tag{OE.3}$$

where from differentiating equation (OE.1) with respect to n_t, that multiplier satisfies the envelope condition

$$J_t = \bar{x}z_t - w_t + \beta(1-\delta)E_t J_{t+1}. \tag{OE.4}$$

Clearly, the last two equations coincide with the free-entry condition and the value of a match, respectively, as they were written in the main text (cf. equations (3) and (1)).

OE.2 Heterogeneous-firm model

Next, suppose there is a representative firm with the capacity to manage a finite, and constant, mass of projects, Υ.[OE.1] In analogy to the setups described in sections 4.1 and OD.1, projects can be either productive, when matched with a worker, or idle. To be precise, we show the representative-firm representation exists for the model with infinitely-lived entrepreneurs discussed in appendix OD.1. The reason is that the assumption analogous to entrepreneur "death" for projects would seem less appealing; it would imply that the firm shrinks to zero over time.[OE.2] Thus, suppose that projects that were previously matched with a worker become again unmatched projects following an exogenous separation shock, which occurs at a rate δ. The probability that such an idle project becomes potentially productive again, so that the firm draws a new idiosyncratic productivity value for it and can potentially match it with a worker, is $1-\gamma$.

[OE.1] That is, one may think of there being a unit continuum of identical firms that each have the same, finite and constant mass of projects Υ, of which a fraction n_t is matched with a worker in a given period, and which take the wage and hiring probability as given.

[OE.2] The proof of an equivalence proposition for the model with "death" involves somewhat lengthy and tedious algebra; it is available on request.

Given this setup, the value function for a firm with n_{t-1} employees then is

$$v(n_{t-1},A_{t-1},z_t,s_{t-1}) = \max_{\hat{a}_t,A_t,n_t} \{(1-\omega)(\bar{x}(n_t z_t + A_t) - n_t \chi) - \kappa[\Gamma_t(1-F(\hat{a}_t))] + \beta E_t[v(n_t,A_t,z_{t+1},s_t))]\},$$

subject to

$$n_t = h_t \Gamma_t (1-F(\hat{a}_t)) + (1-\delta)n_{t-1},$$
$$A_t = (1-\delta)A_{t-1} + h_t \Gamma_t (1-F(\hat{a}_t))a_t^*,$$

with

$$a_t^* = E[a|a \geq \hat{a}] = \frac{1}{1-F(\hat{a}_t)} \int_{\hat{a}_t}^{\bar{a}} a dF(a),$$

and

$$\Gamma_t = \Upsilon - (1-\delta)n_{t-1} - \delta \gamma s_{t-1},$$

and

$$s_t = n_t + \gamma s_{t-1},$$

Notice that, as in the main text, we imposed the linear wage rule. Further, A_t is equal to the total contribution of the idiosyncratic components to firm output. In other words, it is equal to n_t times the average of the idiosyncratic values of all the projects the firm operates.

The first order condition for the cutoff level of project-specific productivity, $\hat{a}_t$, is

$$[(1-\omega)(\bar{x}z_t-\chi)+\beta E_t[v_n(n_t,A_t,z_{t+1},s_t)]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]]\frac{\partial n_t}{\partial \hat{a}_t}$$
$$+[(1-\omega)\bar{x}+\beta E_t[v_A(n_t,A_t,z_{t+1},s_t))]]\frac{\partial A_t}{\partial \hat{a}_t}+\kappa\Gamma_t f(\hat{a}_t)=0,$$

with

$$\frac{\partial n_t}{\partial \hat{a}_t}=-h_t\Gamma_t f(\hat{a}_t), \quad \frac{\partial A_t}{\partial \hat{a}_t}=-h_t\Gamma_t f(\hat{a}_t)a_t^*+h_t\Gamma_t(1-F(\hat{a}_t))\frac{\partial a_t^*}{\partial \hat{a}_t},$$

and

$$\frac{\partial a_t^*}{\partial \hat{a}_t}=\frac{f(\hat{a}_t)}{1-F(\hat{a}_t)}(a_t^*-\hat{a}_t).$$

That is,

$$\begin{aligned}\frac{\partial A_t}{\partial \hat{a}_t}&=-h_t\Gamma_t f(\hat{a}_t)a_t^*+h_t\Gamma_t f(\hat{a}_t)(a_t^*-\hat{a}_t)\\&=-h_t\Gamma_t f(\hat{a}_t)\hat{a}_t.\end{aligned}$$

The first order condition can thus be rewritten as

$$-[(1-\omega)(\bar{x}z_t-\chi)+\beta E_t[v_n(\hat{n}_t,A_t,z_{t+1},s_t)]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]]h_t$$
$$-[(1-\omega)\bar{x}\hat{a}_t+\beta E_t[v_A(n_t,A_t,z_{t+1},s_t)]\hat{a}_t]h_t+\kappa=0. \quad \text{(OE.5)}$$

The envelope condition with respect to A_{t-1} is

$$v_A(n_{t-1},A_{t-1},z_t,s_{t-1})=(1-\omega)\bar{x}(1-\delta)+\beta(1-\delta)E_t[v_A(n_t,A_t,z_{t+1},s_t)]. \quad \text{(OE.6)}$$

The envelope condition with respect to n_{t-1} is

$$v_n(n_{t-1},A_{t-1},z_t,s_{t-1}) = (1-\omega)\left((\bar{x}z_t-\chi)\frac{\partial n_t}{\partial n_{t-1}}+\bar{x}\frac{\partial A_t}{\partial n_{t-1}}\right)+\kappa(1-F(\hat{a}_t))(1-\delta)$$
$$+\beta E_t[v_A(n_t,A_t,z_{t+1},s_t)]\frac{\partial A_t}{\partial n_{t-1}}+\beta E_t[v_n(n_t,A_t,z_{t+1},s_t)]\frac{\partial n_t}{\partial n_{t-1}}$$
$$+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]\frac{\partial s_t}{\partial n_{t-1}}, \quad \text{(OE.7)}$$

with

$$\frac{\partial n_t}{\partial n_{t-1}} = (1-\delta)(1-h_t(1-F(\hat{a}_t))), \quad \text{and} \quad \frac{\partial A_t}{\partial n_{t-1}} = -(1-\delta)h_t(1-F(\hat{a}_t))a_t^*, \qquad \text{(OE.8)}$$

as well as

$$\frac{\partial s_t}{\partial \hat{n}_{t-1}} = (1-\delta)(1-h_t(1-F(\hat{a}_t))). \qquad \text{(OE.9)}$$

Lastly, the envelope condition with respect to s_{t-1} is

$$v_s(n_{t-1},A_{t-1},z_t,s_{t-1}) = (-\delta\gamma)(1-F(\hat{a}_t)\Big(h_t\Big[(1-\omega)(\bar{x}(z_t+a_t^*)-\chi)$$
$$+\beta E_t[v_n(n_t,A_t,z_{t+1},s_t)]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]+\beta E_t[v_A(n_t,A_t,z_{t+1},s_t)]a_t^*\Big]-\kappa\Big)$$
$$+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]\gamma \quad \text{(OE.10)}$$

Inserting the expressions in equations (OE.8) and (OE.9) into equation (OE.7) gives

$$v_n(n_{t-1},A_{t-1},z_t,s_{t-1}) = (1-\delta)\Big\{(1-\omega)\left((\bar{x}z_t-\chi)(1-h_t(1-F(\hat{a}_t))-h_t(1-F(\hat{a}_t))\bar{x}a_t^*\right)$$
$$+\kappa(1-F(\hat{a}_t))-\beta E_t[v_A(n_t,A_t,z_{t+1},s_t)]h_t(1-F(\hat{a}_t))a_t^*+\beta E_t[v_n(n_t,A_t,z_{t+1},s_t)]$$
$$\times(1-h_t(1-F(\hat{a}_t)))+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)](1-h_t(1-F(\hat{a}_t)))\Big\}. \quad \text{(OE.11)}$$

Rearranging

$$v_n(n_{t-1},A_{t-1},z_t,s_{t-1}) = (1-\delta)\Big\{(1-\omega)(\bar{x}z_t-\chi)-(1-F(\hat{a}_t))\Big[h_t\Big((1-\omega)(\bar{x}(z_t+a_t^*)-\chi)$$
$$+\beta E_t[v_A(n_t,A_t,z_{t+1},s_t)]a_t^*+\beta E_t[v_n(n_t,A_t,z_{t+1},s_t)]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]\Big)-\kappa\Big]$$
$$+\beta E_t[v_n(n_t,A_t,z_{t+1},s_t)]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]\Big\}. \quad \text{(OE.12)}$$

Conjecture that $v_n(n_{t-1},A_{t-1},z_t,\Upsilon_t)$ only depends on z_t and a_t, and that $v_A(n_{t-1},A_{t-1},z_t,\Upsilon_t)$ is a constant. That is $v_n(z_t,a_t)$, and v_A. Then define $W(z,a,a')$ as

$$(1-\delta)W(z,a,a') = v_n(z,a')+v_A a.$$

Using this definition we can rewrite equation (OE.12) as

$$W(z_t,a_t^*,a_t^*) = (1-\omega)(\bar{x}(z_t+a_t^*)-\chi)-(1-F(\hat{a}_t))\Big[h_t\Big((1-\omega)(\bar{x}(z_t+a_t^*)-\chi)$$
$$+\beta(1-\delta)E_t[W(z_{t+1},a_t^*,a_{t+1}^*)]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]\Big)-\kappa\Big]$$
$$+\beta(1-\delta)E_t[W(z_{t+1},a_t^*,a_{t+1}^*)]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]. \quad \text{(OE.13)}$$

Define $J_t(a)$ as

$$J_t(a) = (1-\omega)(\bar{x}(z_t+a)-\chi)+\beta(1-\delta)E_t[J_{t+1}(a)]+\beta\delta E_t[\hat{J}_{t+1}^U], \quad \text{(OE.14)}$$

J_t^U as

$$J_t^U = J_t(a)-W(z,a,a').$$

and $\hat{J}_t^U$ (implicitly) as

$$v_s(n_{t-1},A_{t-1},z_t,\Upsilon_t) = \delta(\hat{J}_t^U-J_t^U). \quad \text{(OE.15)}$$

Using these relations we can rewrite equation (OE.13) as

$$J_t(a_t^*) - J_t^U = (1-\omega)(\bar{x}(z_t + a_t^*) - \chi) - (1 - F(\hat{a}_t))\Big[h_t\Big((1-\omega)(\bar{x}(z_t + a_t^*) - \chi) + \beta(1-\delta)E_t[J_{t+1}(a_t^*) - J_{t+1}^U] + \beta\delta E_t[\hat{J}_{t+1}^U - J_{t+1}^U]\Big) - \kappa\Big] + \beta(1-\delta)E_t[J_{t+1}(a_t^*) - J_{t+1}^U] + \beta\delta E_t[\hat{J}_{t+1}^U - J_{t+1}^U],$$

or simply

$$J_t^U = (1 - F(\hat{a}_t))\Big[h_t((1-\omega)(J_t - \beta E_t[J_{t+1}^U]) - \kappa\Big] + \beta E_t[J_{t+1}^U]. \qquad \text{(OE.16)}$$

Moreover equation (OE.10) can be rewritten as

$$v_s(n_{t-1}, A_{t-1}, z_t, s_{t-1}) = (-\delta\gamma)(1 - F(\hat{a}_t))\Big[h_t((1-\omega)(J_t - \beta E_t[J_{t+1}^U]) - \kappa\Big] + \beta E_t[v_s(n_t, A_t, z_{t+1}, s_t)]\gamma,$$

or

$$v_s(n_{t-1}, A_{t-1}, z_t, s_{t-1}) = -\delta\gamma J_t^U + \beta\delta\gamma E_t[J_{t+1}^U] + \beta E_t[v_s(n_t, A_t, z_{t+1}, s_t)]\gamma.$$

Thus

$$\hat{J}_t^U = (1-\gamma)J_t^U + \gamma\beta E_t[\hat{J}_{t+1}^U]. \qquad \text{(OE.17)}$$

Lastly, using the same definitions, we can rewrite the first order condition as

$$\big[(1-\omega)(\bar{x}(z_t + \hat{a}_t - \chi) + \beta(1-\delta)E_t[W(z_{t+1}, \hat{a}_t, a_{t+1}^*)] + \beta E_t[v_s(n_t, A_t, z_{t+1}, s_t)]\big]\, h_t - \kappa = 0,$$

or

$$\kappa = h_t\left[(1-\omega)(\bar{x}(z_t+\hat{a}_t-\chi)+\beta(1-\delta)E_t[W(z_{t+1},\hat{a}_t,a^*_{t+1})]+\beta E_t[v_s(n_t,A_t,z_{t+1},s_t)]\right],$$
$$= h_t\left[(1-\omega)(\bar{x}(z_t+\hat{a}_t-\chi)+\beta(1-\delta)E_t[J_{t+1}(\hat{a}_t)-J^U_{t+1}]+\beta\delta E_t[\hat{J}^U_{t+1}-J^U_{t+1}]\right],$$
$$= h_t(J_t(\hat{a}_t)-\beta E_t[J^U_{t+1}]). \tag{OE.18}$$

Equations (OE.14), (OE.16), (OE.17), and (OE.18) corresponds to the main equations in section OD.1. Thus, the framework can be interpreted either as an infinitely-lived representative firm or as firm-worker pairs.

As indicated already in the opening paragraph of this section, we have shown that a representative, multi-worker firm representation is admissable for the model analyzed in section OD.1 of this online appendix. Relative to the model considered in the main text, equation (OE.14) includes a continuation term $\beta\delta E_t[\hat{J}^U_{t+1}]$ that is absent in the main text (cf. equation (15)) and which reflects the fact that projects continue to have a positive value upon separation. Section OD.1 demonstrated that for $\gamma \to 1$, the two models imply qualitatively and quantitatively similar impulse responses for an uncertainty shock. An exact equivalence to equation (15) can be derived by supposing that a fraction of filled projects, upon exogenous destruction, becomes obsolete and is lost to the operating firm forever; a different, new firm then has the opportunity to launch an equal measure of projects.[OE.3]

[OE.3] As already mentioned in footnote OE.2, the proof is quite lengthy but is available upon request.

Appendix OF Robustness exercises

This appendix section reports the results of several robustness checks regarding the novel search-and-matching (SaM) model with a finite mass of entrepreneurs and firm-specific productivity.

OF.1 Stochastic vs. non-stochastic hiring specification

The model in section 4.1 assumes that in any period t, an unmatched entrepreneur can post a single vacancy, which costs κ units of final goods. The vacancy turns into a match with probability h_t. Otherwise the vacancy remains unfilled and the entrepreneur faces a new decision about whether to remain on the sidelines or post a vacancy at the beginning of the following period. We will refer to this specification as "stochastic hiring." As it is probably the most common way of formulating the SaM model, used in particular also by Leduc and Liu (2016, esp. their eqn. 22), we adopt it as our baseline. An alternative specification supposes that the entrepreneur posts $1/h_t$ vacancies, each costing κ units of the final good, and then creates one job with certainty. We will refer to this case as "non-stochastic hiring."

In the canonical SaM model with risk-neutral entrepreneurs, the two specifications yield isomorphic equilibrium conditions; only the suggested interpretations potentially differ. Either way, in equilibrium, labor market tightness — and, hence, h_t — adjusts such that the expected profit from a new job is equal to the expected cost of hiring a worker. The value of waiting is zero by construction. Under stochastic hiring, it is most natural to write $h_t J_t = \kappa$, meaning that in equilibrium the value of a match discounted by the probability of a vacancy being filled is equal to the fixed cost of posting that vacancy. Under non-stochastic hiring, writing $J_t = \kappa(1/h_t)$ is more natural, in which case equilibrium requires the value of a match to be equal to the expected cost of posting a vacancy until it is filled.

In our modified framework, entrepreneurs are also risk neutral. Nevertheless, the quantitative model outcomes are somewhat different for these two different hiring specifications. In brief, the reason is that under stochastic hiring, expected profits from waiting now and posting a vacancy later are affected by the hiring probability covarying negatively with the match value; the same is not true under non-stochastic hiring. We next discuss these properties in some more detail.

The intuition is most easily described with reference to the two-period model. Recall that in our baseline, the indifference condition is (cf. equation (OC.6))

$$-\kappa + h_1(1+\widehat{a}_1) = E_1\left[p_2(-\kappa + h_2(z_2 + a_2^*))\right].$$

In contrast, when entrepreneur can post multiple vacancies, the condition is

$$-\kappa/h_1 + (1+\widehat{a}_1) = E_1\left[p_2(-\kappa/h_2 + (z_2 + a_2^*))\right].$$

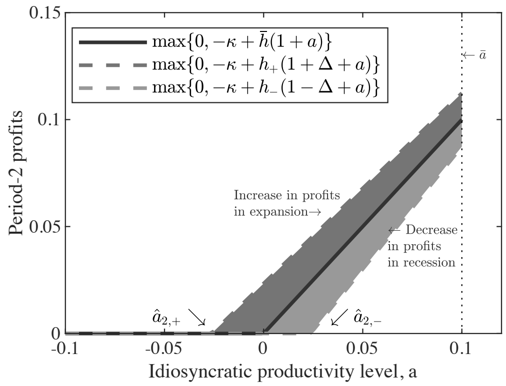
Now, the level of idiosyncratic productivity above which an entrepreneur would rather enter than wait (i.e., not produce at all in the two-period setting), $\widehat{a}_2$, and, hence, the conditional expected value of a draw, a_2^*, are the same across the alternative hiring cost specifications. Instead, the key difference between these two equations is that in the former case, the *expectation* on the right-hand side, conditional on period-1 information, includes a product term involving h_2 and z_2, so that their negative co-movement lowers the value of waiting. By contrast, since under non-stochastic hiring $1/h_2$ multiplies a term that is invariant to uncertainty regarding z_2 (viz., the fixed costs κ), this same mechanism is not operative.[OF.1]

The upper row of panels in Figure OF.1 uses the graphical apparatus set out in section OC to describe the value of waiting due to aggregate uncertainty under the two alternative specifications. As is clearly visible, the expected value of waiting – the difference between the increase in profits in an expansion and the decrease in profits in a recession when integrating over all possible productivity draws above the respective cutoffs – is greater under the alternative, non-stochastic hiring specification. The reason is that the benefit from a higher idiosyncratic productivity draw in an expansion is not scaled down by greater congestion that lowers the probability of that draw

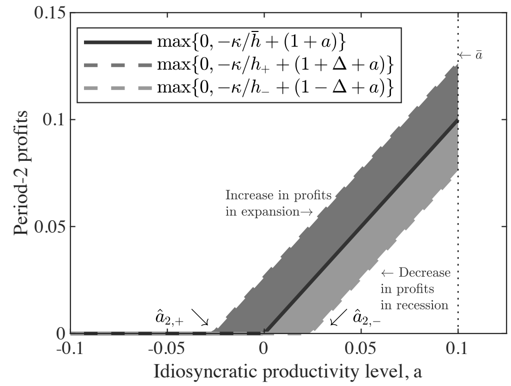
[OF.1] For completeness, notice that the division of κ by h_2 under non-stochastic hiring affects the expected value of hiring costs in the presence of aggregate uncertainty. We consider a quantitative assessment of the two specifications below.

Figure OF.1: Stochastic vs. non-stochastic hiring specification

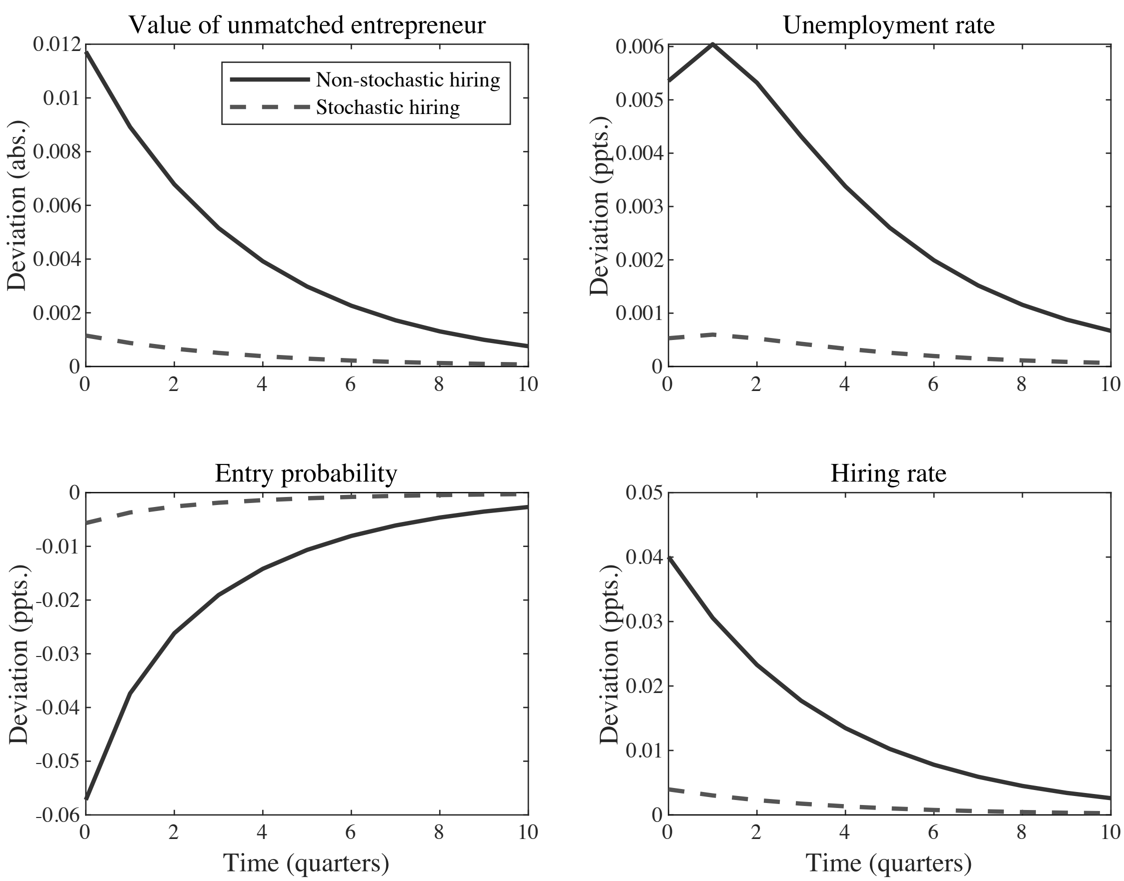
(a) 2-period model: stochastic hiring



(b) 2-period model: non-stochastic hiring



(c) Pure uncertainty IRFs in the full model, $\sigma_a = 0.001$



Notes: Panels **??** and OF.1b show how the value of waiting is determined in a stylized, two-period setting. The left panel corresponds to Figure OC.1c and relies on the "stochastic hiring" assumption. The right panel instead captures the "non-stochastic" specification. To make the visual comparison straightforward, the steady-state hiring rate, h, is set equal to one; the no-aggregate-uncertainty line is then the same for both specifications. For the full, infinite-horizon model, Panel OF.1c plots the usual pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$.

actually yielding profits in a filled match. The opposite holds in a recession, but then these relatively higher probability of hiring boosts expected profits only over a smaller range of productivity draws. (This is the flipside of the argument presented in section OC.2, of course.) Thus, the comparison of alternative hiring specifications informs us that firm size potentially matters for the transmission of uncertainty shocks even under constant returns to scale in production, insofar as the option-value effect will be stronger when an entrepreneur can post several vacancies instead of just one.

How do these different specifications shape the predictions of the full, infinite-horizon model for the effects on economic activity of an anticipated increase in volatility?[OF.2] The solid line in Figure OF.1c graphs the pure uncertainty IRFs for the value of an unmatched entrepreneur, unemployment, the entry probability and the hiring rate, when $\sigma_a = 0.001$ for the non-stochastic hiring specification. The dashed line represents the stochastic hiring benchmark. In either model, the recalibration procedure described in section 4.2 is applied, which facilitates comparison. Consistent with the reasoning developed in the context of the two-period model, the uncertainty effects are several magnitudes larger given the non-stochastic hiring specification than in the baseline. The analysis thus indicates that the results in the main text, which are based on the stochastic hiring assumption, represent a conservative, lower bound for the effects of uncertainty shocks.

[OF.2] To be explicit, under either specification the equation pinning down the cutoff level $\widehat{a}$ for which an entrepreneur is indifferent between going to the matching market is

$$-\kappa/h_t = J_t(\widehat{a}_t) - \beta E_t[J^U_{t+1}].$$

But under the non-stochastic hiring specification the beginning-of-period t of an unmatched entrepreneur is given by

$$J^U_t = \beta E_t[J^U_{t+1}] + p_t\left(-\frac{\kappa}{h_t} + (J_t(a^*_t) - \beta E_t[J^U_{t+1}])\right),$$

with steady-state value $J^U = \frac{p(-\kappa/h+J(a^*))}{1-\beta(1-p)}$. Previously, we had

$$J^U_t = \beta E_t[J^U_{t+1}] + p_t\left(-\kappa + h_t(J_t(a^*_t) - \beta E_t[J^U_{t+1}])\right),$$

and $J^U = \frac{p(-\kappa+h*J(a^*))}{1-\beta(1-ph)}$.

OF.2 Normal distribution

Our baseline specification of the model with heterogeneity in entrepreneurial productivity assumes that firm-level productivity has a uniform distribution in the interval $[-\sqrt{3}\sigma_a, \sqrt{3}\sigma_a]$. Methodologically, our baseline assumption has the advantage that in the recalibration step (cf. section 4.2) we can analytically solve for the elasticity of labor market tightness with respect to productivity. In addition, the uniform distribution ensured consistency between the full model and the analytical, two-period model presented in section OC. This appendix section shows that the results are highly comparable when, instead, firm-level productivity draws comes from a normal distribution, that is, when $a \sim \mathcal{N}(0, \sigma_a^2)$.

Focusing on the implications of firm-level dispersion in the absence of aggregate uncertainty first, the solid lines in the upper panels of Figure OF.2 report the steady-state value of an unmatched entrepreneur, J^U, as well as the unemployment rate as a function of σ_a. To ease comparison, the dashed line repeats the results under a uniform distribution. Unsurprisingly, the qualitative patterns are the same. The only model equations that are directly affected by functional choices for $F(a)$ are the expressions for $p = \text{prob}[a \geq \widehat{a}]$ and $a^* = E[a \geq \widehat{a}]$. In either case, greater values of σ_a increase the value of remaining unmatched, as doing so preserves the option of getting another, better draw in the future. Without recalibrating the model parameters, this effect lowers lower vacancy posting and, hence, increases steady-state unemployment.[OF.3]

Next, Figure OF.2b reports the IRFs for an increase in anticipated future aggregate volatility under the two alternative assumptions. Again the results are highly comparable; the effect sizes are just slightly more pronounced under the assumption of a uniform distribution. These quantitative differences further illustrate the point made in section 4.1 that aggregate uncertainty shocks have a smaller impact for larger values of σ_a when the hiring rate is endogenous.[OF.4] With a 'tighter' distribution, here in the sense of the normal distribution having greater kurtosis than the uniform,

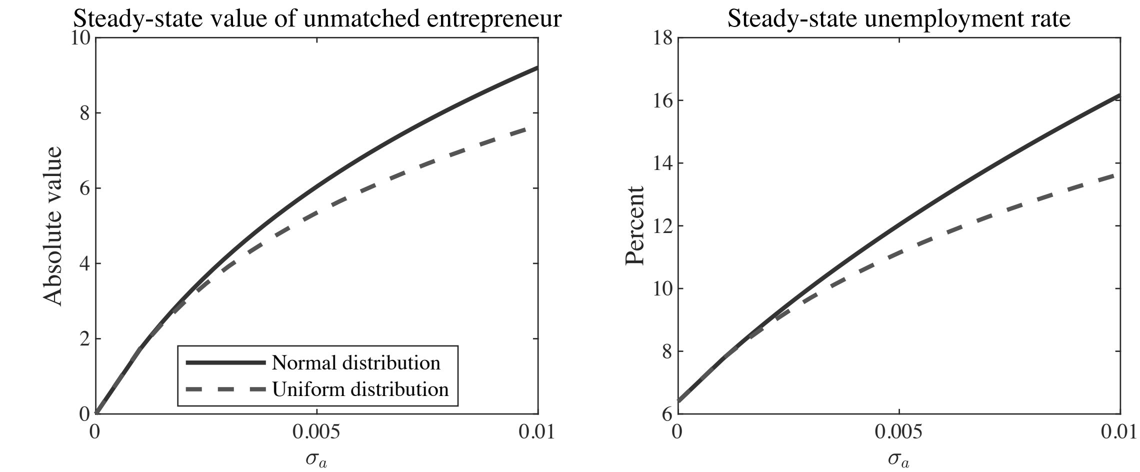
[OF.3] Quantitatively, the strength of this mechanism increases slightly more with σ_a when assuming a normal distribution. For even though idiosyncratic uncertainty raises the conditional expected value of a draw by more under a uniform distribution, the mass of marginal entrepreneurs (i.e., near the indifference point) is greater under a normal distribution.

[OF.4] This result assumes, of course, that in steady state the cutoff is at or near the center of the normal distribution.

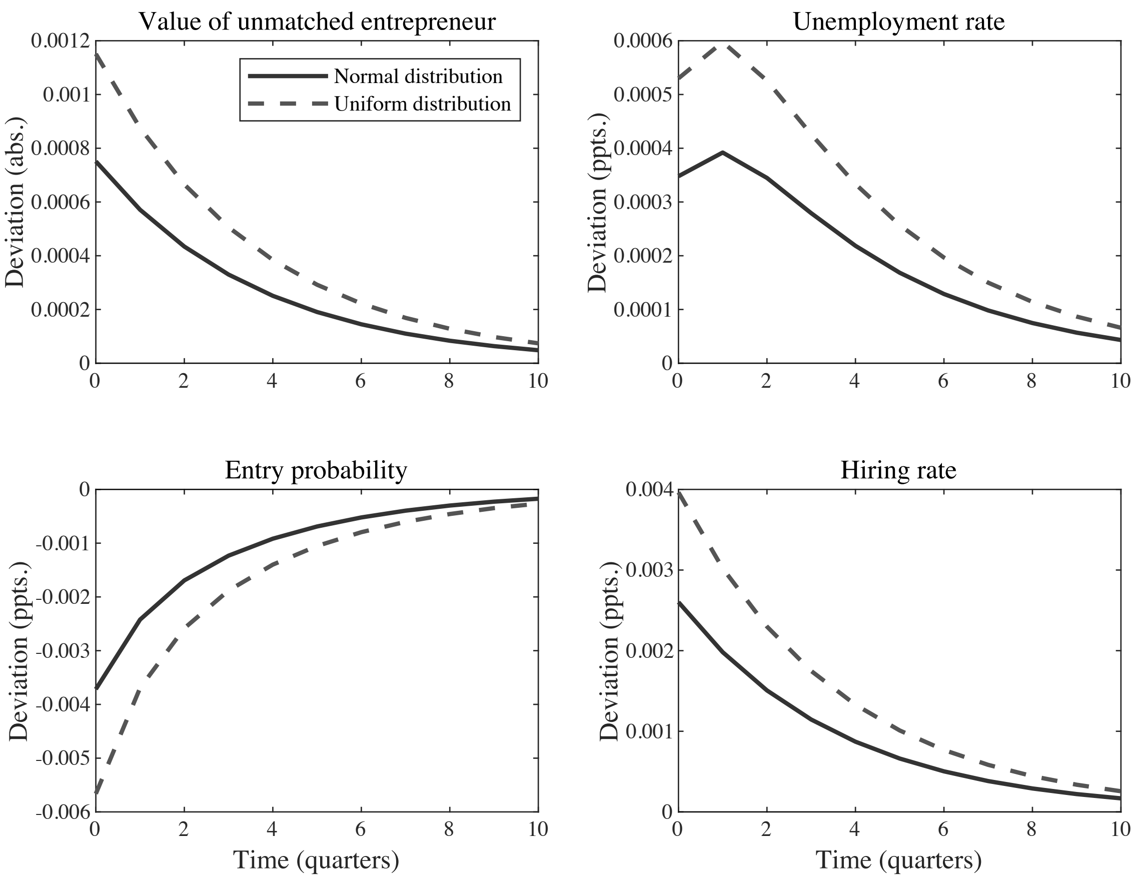
entrepreneurs are more similar. Accordingly, we are closer to the benchmark case where profits are completely eroded due to variations in the hiring rate due to (expected) movements in aggregate productivity.

Figure OF.2: Uncertainty effects with normal distribution for firm-level productivity

(a) Steady-state effects of greater idiosyncratic dispersion



(b) Pure uncertainty IRFs ($\sigma_a = 0.001$)

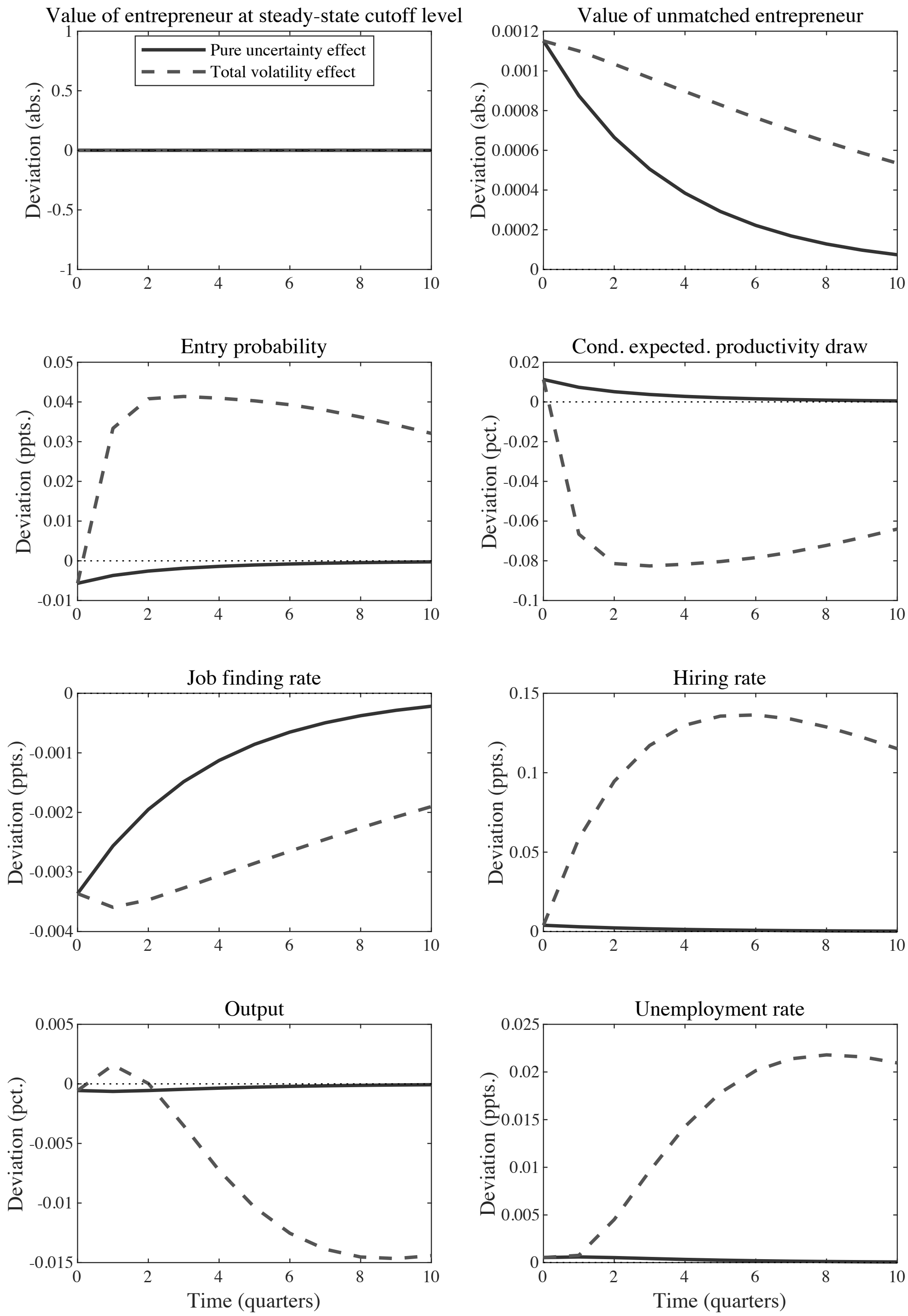


Notes: The two panels in Figure OF.2a describe the steady-state effects of increasing firm-level dispersion, as measured by σ_a, assuming a normal (solid line) or uniform (dashed line) respectively. The panels in Figure OF.2b reports pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$ under these two assumptions, imposing $\sigma_a = 0.001$ and $P = 0.5$. In the latter case, the model parameters are recalibrated following the procedure described in section 4.2.

OF.3 Different degree of cross-sectional dispersion

Figure 5 indicated pure uncertainty and total volatility IRFs conditional on the standard deviation of idiosyncratic productivity shocks, σ_a, being equal to 0.003. Figure OF.3 reports the same IRFs when $\sigma_a = 0.001$ instead. A comparison of the two figures reveals that the magnitude of uncertainty effects is generally larger for the higher value of σ_a, in line with what is shown in figure 5 for the unemployment rate, specifically. Interestingly, the composition effect – following a volatility shock the average productivity of active firms initially rises – is now sufficiently strong for the total volatility effect on output to reach positive territory, albeit only very briefly, before the negative impact of rising unemployment kicks in.

Figure OF.3: Uncertainty effects in SaM model with cross-sectional dispersion; $\sigma_a = 0.001$



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The model parameters are recalibrated following the procedure described in section 4.2. In particular, $\chi = 0.682$, $\omega = 0.885$, and $\bar{z} = 0.999$.

OF.4 Different steady-state entry probability

In the baseline analysis, we calibrated the model such that in steady state half of the available entrepreneurs actually post a vacancy. As noted in the main text, this choice has a number of practical advantages. For completeness, we report the results of a robustness exercise in which the steady-state value for p is 0.2 instead of 0.5.

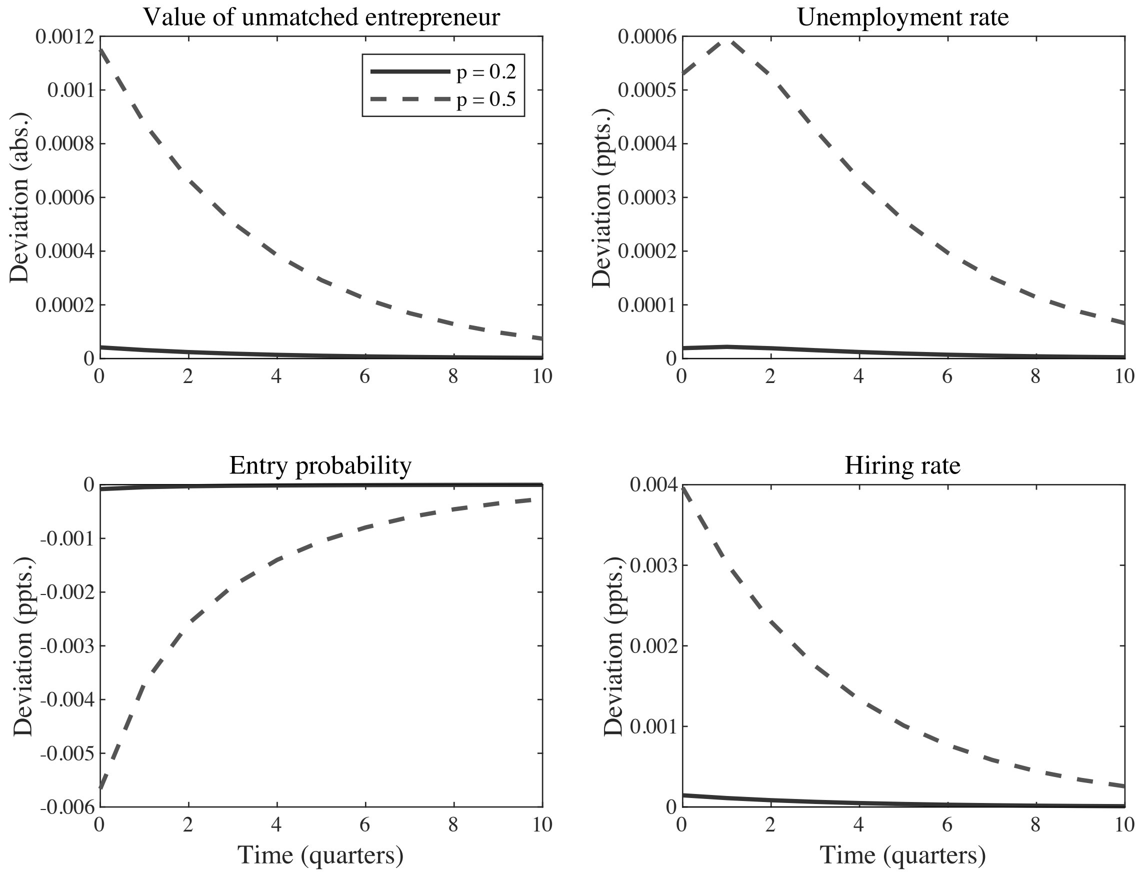
Figure OF.4 compares the pure uncertainty IRFs for the value of an unmatched entrepreneur and unemployment, as well as the entry probability and the hiring rate, when $\sigma_a = 0.001$ for both $p = 0.2$ (solid line) and $p = 0.5$ (dashed line; as in figure OF.3).[OF.5] Clearly, the value of waiting due to greater anticipated uncertainty rises by less in the latter case, and accordingly unemployment worsens by less also. The reason is that a lower steady-state entry probability is associated with a larger mass of potential entrepreneurs, Υ. As such, for any given degree of firm-level dispersion we are closer to the benchmark SaM model with an infinite mass of potential entrepreneurs. In that latter model, the option value of waiting, and thus also pure uncertainty effects given risk neutrality and linear wages, are zero. In more economic terms, with more potential entrepreneurs, adjustments in the hiring rate in both good and bad productivity states are sufficient to leave expected profits nearly invariant to changed in anticipated volatility.

Imposing a lower steady state probability of entry means that the admissable range of σ_a is wider when recalibrating the model as described in section 4.2 (as the worker bargaining power parameter ω hits the lower limit of zero for higher values of σ_a when p is lower, that is, when the mass of potential entrepreneurs is greater). In this spirit, the solid line in Figure OF.4b indicates the impact effect of an uncertainty shock as a function of σ_a when $p = 0.2$. The dashed line refers to the same statistic for a fixed value $\sigma_a = 0.001$ but with $p = 0.5$. This analysis makes clear that for a lower steady-state entry probability, the option-value effect due to anticipated greater future volatility matches that obtained with a higher steady-state entry probability for a greater degree of firm-level dispersion.

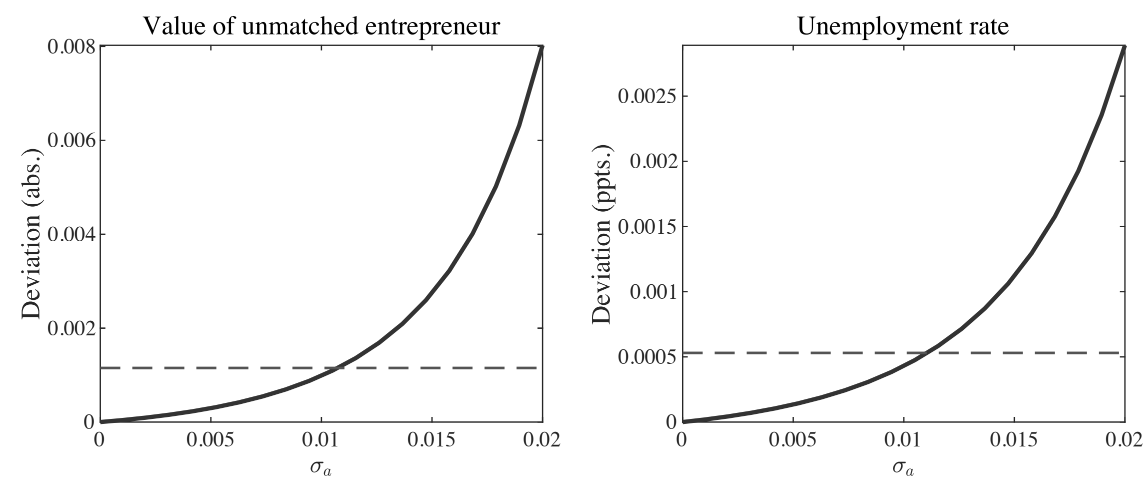
[OF.5] Given our recalibration procedure, steady-state unemployment is the same across the different specifications.

Figure OF.4: Uncertainty effects with lower steady-state entry probability

(a) Pure uncertainty IRFs ($\sigma_a = 0.001$)



(b) Impact effects of uncertainty shock for different σ_a values ($p = 0.2$)



Notes: The upper panel reports pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$, assuming that $\sigma_a = 0.001$ and that the steady-state entry probability is either $p = 0.2$ (solid line) or $p = 0.5$ (dashed line). The bottom panel plots the impact effect in the case of $p = 0.2$ as a function of σ_a (solid line). Here, the dashed line describes the impact effect for a fixed value of $\sigma_a = 0.001$ and with $p = 0.5$. The model parameters are recalibrated following the procedure described in section 4.2.

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